A prospect for super resolution [a]

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INTRODUCTION

Wouldn't it be great if I could take signals of 10-30 Hz bandwidth from 100 different offsets and construct a zero-offset trace with 5-100 Hz bandwidth? This would not violate Shannon's sampling theorem which theoretically allows us to have a transform from 100 signals of 20 Hz bandwidth to one signal at 2000 Hz bandwidth. The trouble is that simple NMO is not such a transformation. Never-the-less, if the different offsets really did give us any extra information, we should be able to put the information into extra bandwidth. Let us consider noise free synthetic data and see if we can come up with a model where this could happen.

FITTING FRAMEWORK

The operator of interest is the one that creates many offsets of seismic data from a zero-offset model space.

$\mathbf{z}$ is a white seismic trace (model) at zero-offset

$\mathbf{d}_j$ is a red seismic trace (data) at nonzero-offset x_j

$\mathbf{L}$ is a seismic band pass filter

$\mathbf{H}_j$ sprays along hyperbola using a known, rough $v(z)$

$\overline{\mathbf{H}}_j$ sprays using a known, smooth $\overline{v}(z)$

The operator of interest is the one that transforms $\mathbf{z}$ to all the data $\mathbf{d}_j$ at all of the offsets x_j.

$$\mathbf{d}_j \quad = \quad \mathbf{L}\mathbf{H}_j\mathbf{z} \tag{1}$$

Here is a trivial idea: Estimates $\hat{\mathbf{z}}_j$ of $\mathbf{z}$ from data $\mathbf{d}_j$ at different offsets x_j have different spectral bands because of NMO stretch. Wide offsets create low frequency. Trouble is, these low frequencies add little spectral bandwidth. We want extra high frequencies too.

We know a simple two-step process where one offset can be obtained from another: First moveout for one offset. Then inverse moveout for the other offset. Whenever such offset continuation works, extra offsets cannot bring us extra information. Extra traces give only redundancy.

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Inversion theory says if the transformation has no null space we should be able to solve for everything. Since in practice we cannot seem to obtain that extra bandwidth, it seems that the operator $\mathbf{LH}_j$ has a large null space, about equal in size to the trace length times (the number of offsets minus one).

ROUGH VELOCITY(Z)

Taking velocity to be a rough (bumpy) function of depth, different offset traces might be fundamentally different thus providing different information (i.e. more information hence potentially more bandwidth). The bumpy velocity model seems artificial since it requires to be known a rough velocity as a function depth. Never-the-less, the idea could be helpful because we sometimes have well logs, or we might later learn how to bootstrap our velocity estimate from a smooth velocity to a rougher one.

Many people think about rough *impedance.* Here we consider a rough *interval velocity.*

ROUGH V(Z) MAKES TAU(T) MULTIVALUED

According to the Dix approximation, travel time $t(\tau)$ is a unique function of vertical travel time τ because

$$t^2 \quad = \quad \tau^2 + x^2/v(\tau)^2 \tag{2}$$

The reverse is not true, however, $\tau(t)$ can be a multivalued function of t, and is especially likely to be so where $v(\tau)$ is a rough function of τ. When $\tau(t)$ is a multivalued function of t the process of offset continuation breaks down. Then extra offsets are providing extra information. We don't yet know if the extra information is a small amount or a large amount or whether that extra information is uniformly or locally distributed. Figure 1 shows an example.

Figure 1 shows two kinds of multivaluedness in the transformation. First is the familiar kind that arises whenever $dv/dz > 0$ where travel times of shallow waves cross those of deep waves. Let us place a line through the broad maxima in $t(\tau)$ at about $t = 2.5\tau$ for all x. In a constant velocity earth, the ratio $t/\tau = 2.5$ corresponds to a propagation angle $\cos\theta = \tau/t$ or about $\theta = 66^\circ$. Thus, a wave with average angle greater than $\theta = 66^\circ$ generally arrives at the same time and offset as another wave with an average angle less than $\theta = 66^\circ$.

The second way of being multivalued is less familiar and hence more interesting, the roughness in the $t(\tau)$ transformation. We see this roughness does give rise to multivaluedness. Disappointingly, the multivaluedness is not found everywhere but mainly along the $\theta = 66^\circ$ trend. We have not yet answered how much extra information we can obtain from this. Clearly though, if multivaluedness is what makes different offsets give us different information, it is along this "mute-line" $\theta = 66^\circ$ trend where we must look.

Let us find the high frequency. Where does an observable (low) frequency on the t axis map to a high frequency on the τ axis? It happens where a long region on the t axis maps to a short region on the τ axis, in other words, where the slope $dt/d\tau$ is greatest. This is the opposite of usual NMO in the neighborhood of the diagonal asymptote in Figure 1 where $dt/d\tau < 1$. From the figure, we see the possibility for frequency boosting does not arise from the roughness in velocity but just beneath the water bottom at any offset, i.e.,

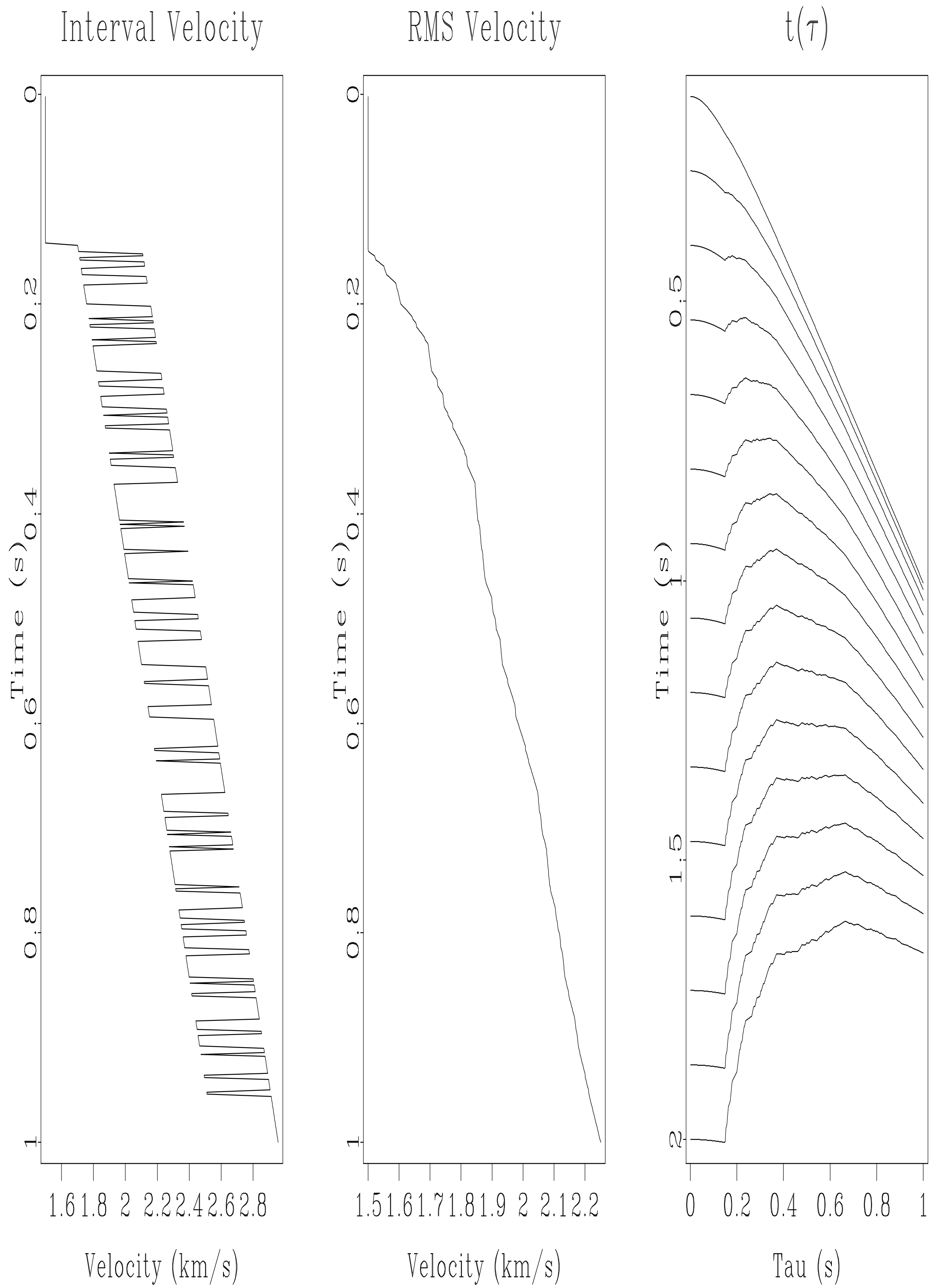

Figure 1: Right shows $t(\tau)$ for many offsets.

at the greatest angles. Since $dt/d\tau$ is negative there, it gives a kind of upside-down image. To understand this image, think of head waves where the deepest layer is fastest and hence has the earliest arrival with *shallower* layer arrivals coming *later*.

It is possible the Dix approximation is breaking down here, a concern that requires further study. Accurate *reflection* seismograms in this region are easy to make with the phase shift method. Getting correct head waves is more complicated.

FURTHER STEPS

Each offset x_j allows us to make a different estimate of the earth model $\mathbf{z}_j$. There are two possiblities:

i.e. $\mathbf{z}_j \neq \mathbf{z}_{j+1}$.

$$\hat{\mathbf{z}}_j = \mathbf{H}_j' \mathbf{L}' \mathbf{d}_j \tag{3}$$

$$\hat{\hat{\mathbf{z}}}_j = \overline{\mathbf{H}}_j' \mathbf{L}' \mathbf{d}_j \tag{4}$$

We should plot $\hat{\mathbf{z}}_j$ as a function of j. We should also plot $\hat{\mathbf{z}}_j - \hat{\mathbf{z}}_{j-1}$ as a function of j and see if we can find any higher temporal frequencies.