

Nonhyperbolic reflection moveout of P -waves: An overview and comparison of reasons^a

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ABSTRACT

The familiar hyperbolic approximation of P -wave reflection moveout is exact for homogeneous isotropic or elliptically anisotropic media above a planar reflector. Any realistic combination of heterogeneity, reflector curvature, and nonelliptic anisotropy will cause departures from hyperbolic moveout at large offsets. Here, we analyze the similarities and differences in the influence of those three factors on P -wave reflection traveltimes. Using the weak-anisotropy approximation for velocities in transversely isotropic media with a vertical symmetry axis (VTI model), we show that although the nonhyperbolic moveout due to both vertical heterogeneity and reflector curvature can be interpreted in terms of effective anisotropy, such anisotropy is inherently different from that of a generic homogeneous VTI model.

INTRODUCTION

The hyperbolic approximation of P -wave reflection traveltimes in common-midpoint gathers plays an important role in conventional seismic data processing and interpretation. It is well known that hyperbolic moveout gives exact traveltimes for homogeneous isotropic or elliptically anisotropic media overlaying a plane dipping reflector. Deviations from this simple model generally cause departure from hyperbolic moveout. If the nonhyperbolicity is measurable, we can take it into account to correct errors in conventional processing or to obtain additional information about the medium. To achieve this, however, it is important to know what causes the P -wave moveouts to be nonhyperbolic. Although seismic anisotropy is one possible reason, it is not always the dominant one; others include the vertical or lateral heterogeneity and reflector curvature. In this paper, we give a theoretical description of P -wave reflection traveltimes in different models and compare the behavior and degree of nonhyperbolic moveout caused by various reasons.

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A transversely isotropic model with a vertical symmetry axis (VTI medium) is the most commonly used anisotropic model for sedimentary basins, where the deviation from isotropy is usually attributed to some combination of fine layering and inherent anisotropy of shales. One of the first nonhyperbolic approximations for the P -wave reflection traveltimes in VTI media was proposed by Muir and Dellinger (1985) and further developed by Dellinger et al. (1993). Thomsen (1986) introduced a convenient parameterization of VTI media that was used by Tsvankin and Thomsen (1994) to describe nonhyperbolic reflection moveouts.

We begin with an overview of the weak-anisotropy approximation for P -wave velocities in VTI media and use it for analytic derivations throughout the paper. First, we consider a vertically heterogeneous anisotropic layer. For this model, we compare the three-parameter approximation for the P -wave traveltimes suggested by Tsvankin and Thomsen (1994) with the shifted hyperbola (Malovichko, 1978; Castle, 1988; de Bazelaire, 1988). Next, we examine P -wave moveout in VTI media above a curved reflector. We analyze the cumulative action of anisotropy, reflector dip, and reflector curvature, and develop an appropriate three-parameter representation for the reflection moveout. Finally, we consider models characterized by weak lateral heterogeneity and show that it can mimic the influence of transverse isotropy on nonhyperbolic moveout.

WEAK ANISOTROPY APPROXIMATION FOR VTI MEDIA

In transversely isotropic media, velocities of seismic waves depend on the direction of propagation measured from the symmetry axis. Thomsen (1986) introduced a notation for VTI media by replacing the elastic stiffness coefficients with the P - and S -wave velocities along the symmetry axis and three dimensionless anisotropic parameters. As shown by Tsvankin (Tsvankin, 1996), the P -wave seismic signatures in VTI media can be conveniently expressed in terms of Thomsen's parameters ϵ and δ . Deviations of these parameters from zero characterize the relative strength of anisotropy. For small values of these parameters, the weak-anisotropy approximation (Thomsen, 1986; Tsvankin and Thomsen, 1994) reduces to simple linearization.

The squared group velocity V_g^2 of P -waves in weakly anisotropic VTI media can be expressed as a function of the group angle ψ measured from the vertical symmetry axis as follows:

$$V_g^2(\psi) = V_z^2 \left(1 + 2\delta \sin^2 \psi \cos^2 \psi + 2\epsilon \sin^4 \psi \right), \quad (1)$$

where $V_z = V_g(0)$ is the P -wave vertical velocity, and δ and ϵ are Thomsen's dimensionless anisotropic parameters, which are assumed to be small quantities:

$$|\epsilon| \ll 1, \quad |\delta| \ll 1. \quad (2)$$

Both parameters are equal to zero in isotropic media.

Equation (1) is accurate up to the second-order terms in ϵ and δ . We retain this level of accuracy throughout the paper. As follows from equation (1), the velocity V_x corresponding to ray propagation in the horizontal direction is

$$V_x^2 = V_g^2(\pi/2) = V_z^2 (1 + 2\epsilon) . \quad (3)$$

Equation (3) is actually exact, valid for any strength of anisotropy. Another important quantity is the normal-moveout (NMO) velocity, V_n , that determines the small-offset P -wave reflection moveout in homogeneous VTI media above a horizontal reflector. Its exact expression is (Thomsen, 1986)

$$V_n^2 = V_z^2 (1 + 2\delta) . \quad (4)$$

If $\delta = 0$ as, for example, in the ANNIE model proposed by Schoenberg et al. (1996), the normal-moveout velocity is equal to the vertical velocity.

It is convenient to rewrite equation (1) in the form

$$V_g^2(\psi) = V_z^2 \left(1 + 2\delta \sin^2 \psi + 2\eta \sin^4 \psi \right) , \quad (5)$$

where

$$\eta = \epsilon - \delta . \quad (6)$$

Equation (6) is the weak-anisotropy approximation for the *anellipticity* coefficient η introduced by Alkhalifah and Tsvankin (1995). For the elliptic anisotropy, $\epsilon = \delta$ and $\eta = 0$. To see why the group-velocity function becomes elliptic in this case, note that for small δ

$$\left. \frac{1}{V_g^2(\psi)} \right|_{\eta=0} = \frac{1}{V_z^2 (1 + 2\delta \sin^2 \psi)} \approx \frac{\cos^2 \psi}{V_z^2} + \frac{(1 - 2\delta) \sin^2 \psi}{V_z^2} \approx \frac{\cos^2 \psi}{V_z^2} + \frac{\sin^2 \psi}{V_n^2} . \quad (7)$$

Seismic data often indicate that $\epsilon > \delta$, so the anellipticity coefficient η is usually positive.

An equivalent form of equation (1) can be obtained in terms of the three characteristic velocities V_z , V_x , and V_n :

$$V_g^2(\psi) = V_z^2 \cos^2 \psi + (V_n^2 - V_x^2) \sin^2 \psi \cos^2 \psi + V_x^2 \sin^2 \psi . \quad (8)$$

From equation (8), in the linear approximation the anelliptic behavior of velocity is controlled by the difference between the normal moveout and horizontal velocities or, equivalently, by the difference between anisotropic coefficients ϵ and δ .

We illustrate different types of the group velocities (wavefronts) in Figure 1. The wavefront, circular in the isotropic case (Figure 1a), becomes elliptical when $\epsilon = \delta \neq 0$ (Figure 1b). In the ANNIE model, the vertical and NMO velocities are equal (Figure 1c). If $\epsilon > 0$ and $\delta < 0$, the three characteristic velocities satisfy the inequality $V_x > V_z > V_n$ (Figure 1d).

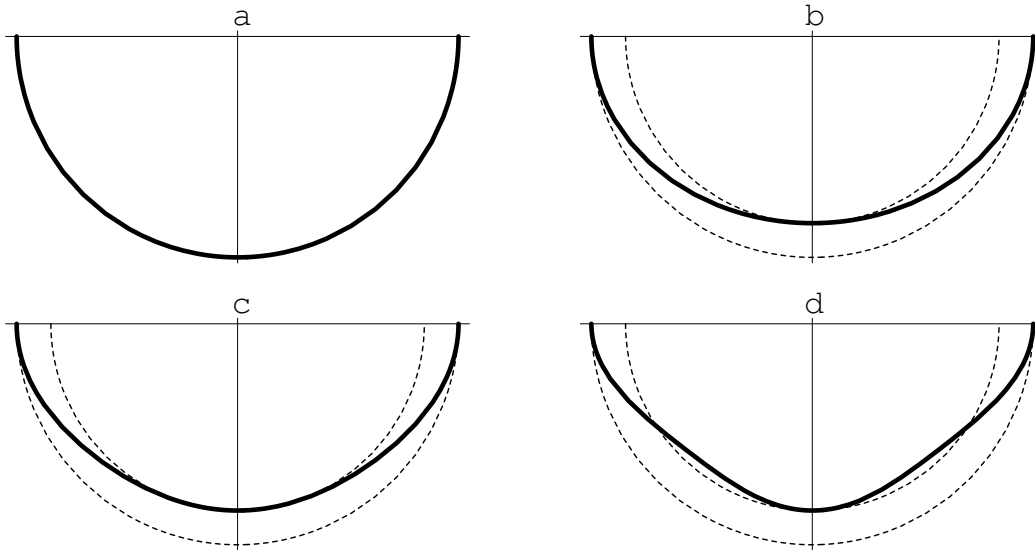


Figure 1: Wavefronts in isotropic medium, $\epsilon = \delta = 0$ (a), elliptically anisotropic medium, $\epsilon = \delta = 0.2$ (b), ANNIE model, $\epsilon = 0.2$, $\delta = 0$ (c), and anisotropic medium with $\epsilon = 0.2$, $\delta = -0.2$ (d). Solid curves represent the wavefronts. Dashed lines correspond to isotropic wavefronts for the vertical and horizontal velocities.

HORIZONTAL REFLECTOR BENEATH A HOMOGENEOUS VTI MEDIUM

To exemplify the use of weak anisotropy, let us consider the simplest model of a homogeneous VTI medium above a horizontal reflector. For an isotropic medium, the reflection traveltime curve is an exact hyperbola, as follows directly from the Pythagorean theorem (Figure 2)

$$t^2(l) = \frac{4z^2 + l^2}{V_z^2} = t_0^2 + \frac{l^2}{V_z^2}, \quad (9)$$

where z denotes the depth of reflector, l is the offset, $t_0 = t(0)$ is the zero-offset traveltime, and V_z is the isotropic velocity. For a homogeneous VTI medium, the velocity V_z in equation (9) is replaced by the angle-dependent group velocity V_g . This replacement leads to the exact traveltimes if no approximation for the group velocity is used, since the ray trajectories in homogeneous VTI media remain straight, and the reflection point does not move. We can also obtain an approximate traveltime using the approximate velocity V_g defined by equations (1) or (5), where the ray angle ψ is given by

$$\sin^2 \psi = \frac{l^2}{4z^2 + l^2}. \quad (10)$$

Substituting equation (10) into (5) and linearizing the expression

$$t^2(l) = \frac{4z^2 + l^2}{V_g^2(\psi)} \quad (11)$$

with respect to the anisotropic parameters δ and η , we arrive at the three-parameter nonhyperbolic approximation (Tsvankin and Thomsen, 1994)

$$t^2(l) = t_0^2 + \frac{l^2}{V_n^2} - \frac{2\eta l^4}{V_n^2 (V_n^2 t_0^2 + l^2)}, \quad (12)$$

where the normal-moveout velocity V_n is defined by equation (4). At small offsets ($l \ll z$), the influence of the parameter η is negligible, and the traveltime curve is nearly hyperbolic. At large offsets ($l \gg z$), the third term in equation (12) has a clear influence on the traveltime behavior. The Taylor series expansion of equation (12) in the vicinity of the vertical zero-offset ray has the form

$$t^2(l) = t_0^2 + \frac{l^2}{V_n^2} - \frac{2\eta l^4}{V_n^4 t_0^2} + \frac{2\eta l^6}{V_n^6 t_0^4} - \dots \quad (13)$$

When the offset l approaches infinity, the traveltime approximately satisfies an intuitively reasonable relationship

$$\lim_{l \rightarrow \infty} t^2(l) = \frac{l^2}{V_x^2}, \quad (14)$$

where the horizontal velocity V_x is defined by equation (3). Approximation (12) is analogous, within the weak-anisotropy assumption, to the “skewed hyperbola” equation (Byun et al., 1989) which uses the three velocities V_z , V_n , and V_x as the parameters of the approximation:

$$t^2(l) = t_0^2 + \frac{l^2}{V_n^2} - \frac{l^4}{V_n^2 t_0^2 + l^2} \left(\frac{1}{V_n^2} - \frac{1}{V_x^2} \right). \quad (15)$$

The accuracy of equation (12), which usually lies within 1% error up to offsets twice as large as reflector depth, can be further improved at any finite offset by modifying the denominator of the third term (Alkhalifah and Tsvankin, 1995; Grechka and Tsvankin, 1998).

Muir and Dellinger (1985) suggested a different nonhyperbolic moveout approximation in the form

$$t^2(l) = t_0^2 + \frac{l^2}{V_n^2} - \frac{f(1-f)l^4}{V_n^2 (V_n^2 t_0^2 + f l^2)}, \quad (16)$$

where f is the dimensionless parameter of anellipticity. At large offsets, equation (16) approaches

$$\lim_{l \rightarrow \infty} t^2(l) = f \frac{l^2}{V_n^2}. \quad (17)$$

Comparing equations (14) and (17), we can establish the correspondence

$$f = \frac{V_n^2}{V_x^2} = \frac{1+2\delta}{1+2\epsilon} \approx 1 - 2\eta. \quad (18)$$

Taking this equality into account, we see that equation (16) is approximately equivalent to equation (12) in the sense that their difference has the order of η^2 .

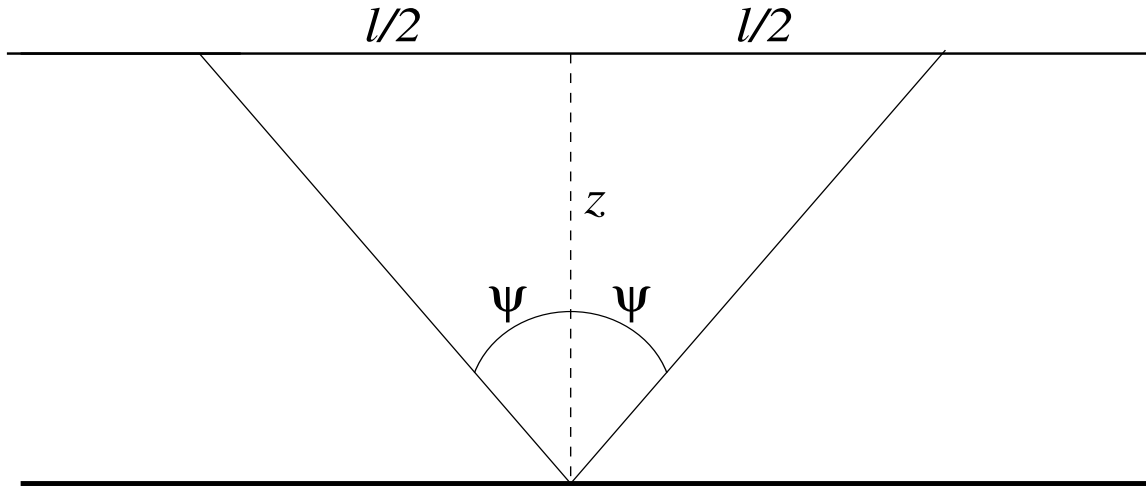


Figure 2: Reflected rays in a homogeneous VTI layer above a horizontal reflector (a scheme).

VERTICAL HETEROGENEITY

Vertical heterogeneity is another reason for nonhyperbolic moveout. We start this section by reviewing well-known results for isotropic media. Although these results can be interpreted in terms of an effective anisotropy, we show that it has different properties than those for the VTI model. We then extend the theory to vertically heterogeneous VTI media and perform a comparative analysis of various three-parameter nonhyperbolic approximations.

Vertically heterogeneous isotropic model

Nonhyperbolicity of reflection moveout in vertically heterogeneous isotropic media has been extensively studied using the Taylor series expansion in the powers of the offset (Bolshykh, 1956; Taner and Koehler, 1969; Al-Chalabi, 1973). The most important property of vertically heterogeneous media is that the ray parameter

$$p \equiv \frac{\sin \psi(z)}{V_z(z)}$$

does not change along any given ray (Snell's law). This fact leads to the explicit parametric relationships

$$t(p) = 2 \int_0^z \frac{dz}{V_z(z) \cos \psi(z)} = \int_0^{t_z} \frac{dt_z}{\sqrt{1 - p^2 V_z^2(t_z)}}, \quad (19)$$

$$l(p) = 2 \int_0^z dz \tan \psi(z) = \int_0^{t_z} \frac{p V_z^2(t_z) dt_z}{\sqrt{1 - p^2 V_z^2(t_z)}}, \quad (20)$$

where

$$t_z = t(0) = 2 \int_0^z \frac{dz}{V_z(z)}. \quad (21)$$

Straightforward differentiation of parametric equations (19) and (20) yields the first four coefficients of the Taylor series expansion

$$t^2(l) = a_0 + a_1 l^2 + a_2 l^4 + a_3 l^6 + \dots \quad (22)$$

in the vicinity of the vertical zero-offset ray. Series (22) contains only even powers of the offset l because of the reciprocity principle: the pure-mode reflection traveltime is an even function of the offset. The Taylor series coefficients for the isotropic case are defined as follows:

$$a_0 = t_z^2, \quad (23)$$

$$a_1 = \frac{1}{V_{rms}^2}, \quad (24)$$

$$a_2 = \frac{1 - S_2}{4 t_z^2 V_{rms}^4}, \quad (25)$$

$$a_3 = \frac{2 S_2^2 - S_2 - S_3}{8 t_z^4 V_{rms}^6}, \quad (26)$$

where

$$V_{rms}^2 = M_1, \quad (27)$$

$$M_k = \frac{1}{t_z} \int_0^{t_z} V_z^{2k}(t) dt \quad (k = 1, 2, \dots), \quad (28)$$

$$S_k = \frac{M_k}{V_{rms}^{2k}} \quad (k = 2, 3, \dots). \quad (29)$$

Equation (24) shows that, at small offsets, the reflection moveout has a hyperbolic form with the normal-moveout velocity V_n equal to the root-mean-square velocity V_{rms} . At large offsets, however, the hyperbolic approximation is no longer accurate. Studying the Taylor series expansion (22), Malovichko (1978) introduced a three-parameter approximation for the reflection traveltime in vertically heterogeneous isotropic media. His equation has the form of a shifted hyperbola (Castle, 1988; de Bazelaire, 1988):

$$t(l) = \left(1 - \frac{1}{S}\right) t_0 + \frac{1}{S} \sqrt{t_0^2 + S \frac{l^2}{V_n^2}}. \quad (30)$$

If we set the zero-offset traveltime t_0 equal to the vertical traveltime t_z , the velocity V_n equal to V_{rms} , and the *parameter of heterogeneity* S equal to S_2 , equation (30)

guarantees the correct coefficients a_0 , a_1 , and a_2 in the Taylor series (22). Note that the parameter S_2 is related to the variance σ^2 of the squared velocity distribution, as follows:

$$\sigma^2 = M_2 - V_{rms}^4 = V_{rms}^4 (S_2 - 1). \quad (31)$$

According to equation (31), this parameter is always greater than unity (it equals 1 in homogeneous media). In many practical cases, the value of S_2 lies between 1 and 2. We can roughly estimate the accuracy of approximation (30) at large offsets by comparing the fourth term of its Taylor series with the fourth term of the exact traveltimes expansion (22). According to this estimate, the error of Malovichko's approximation is

$$\frac{\Delta t^2(l)}{t^2(0)} = \frac{1}{8} (S_3 - S_2^2) \left(\frac{l}{t_0 V_n} \right)^6. \quad (32)$$

As follows from the definition of the parameters S_k [equations (29)] and the Cauchy-Schwartz inequality, the expression (32) is always nonnegative. This means that the shifted-hyperbola approximation tends to overestimate traveltimes at large offsets. As the offset approaches infinity, the limit of this approximation is

$$\lim_{l \rightarrow \infty} t^2(l) = \frac{1}{S} \frac{l^2}{V_n^2}. \quad (33)$$

Equation (33) indicates that the effective horizontal velocity for Malovichko's approximation (the slope of the shifted hyperbola asymptote) differs from the normal-moveout velocity. One can interpret this difference as evidence of some *effective* depth-variant anisotropy. However, the anisotropy implied in equation (30) differs from the true anisotropy in a homogeneous transversely isotropic medium [see equation (1)]. To reveal this difference, let us substitute the effective values $t(l) = \sqrt{4z^2 + l^2}/V_g(\psi)$, $t_0 = 2z/V_z$, $l = 2z \tan \psi$, and $S = V_x^2/V_n^2$ into equation (30). After eliminating the variables z and l , the result takes the form

$$\frac{1}{V_g(\psi)} = \frac{1}{V_z} \left\{ \cos \psi \left(1 - \frac{V_n^2}{V_x^2} \right) + \sqrt{\frac{V_z^2}{V_x^2} \sin^2 \psi + \frac{V_n^4}{V_x^4} \cos^2 \psi} \right\}. \quad (34)$$

If the anisotropy is induced by vertical heterogeneity, $V_x \geq V_n \geq V_z$. Those inequalities follow from the definitions of V_{rms} , t_z , S_2 , and the Cauchy-Schwartz inequality. They reduce to equalities only when velocity is constant. Linearizing expression (34) with respect to Thomsen's anisotropic parameters δ and ϵ , we can transform it to the form analogous to that of equation (5):

$$V_g^2(\psi) = V_z^2 \left[1 + 2\delta \sin^2 \psi + 2\eta(1 - \cos \psi)^2 \right]. \quad (35)$$

Figure 3 illustrates the difference between the VTI model and the effective anisotropy implied by the Malovichko approximation. The differences are noticeable in both the shapes of the effective wavefronts (Figure 3a) and the moveouts (Figure 3b).

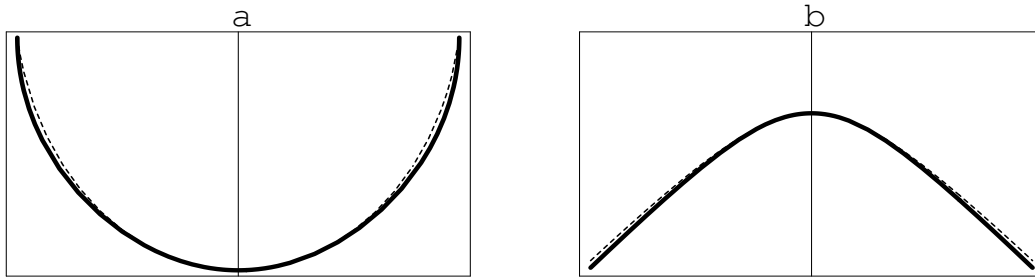


Figure 3: Comparison of the wavefronts (a) and moveouts (b) in the VTI (solid) and vertically inhomogeneous isotropic media (dashed). The values of the effective vertical, horizontal, and NMO velocities are the same in both media and correspond to Thomsen's parameters $\epsilon = 0.2$ and $\delta = 0.1$.

In deriving equation (35), we have assumed the correspondence

$$S = \frac{V_x^2}{V_n^2} = \frac{1 + 2\epsilon}{1 + 2\delta} \approx 1 + 2\eta. \quad (36)$$

We could also have chosen the value of the parameter of heterogeneity S that matches the coefficient a_2 given by equation (25) with the corresponding term in the Taylor series (13). Then, the value of S is (Alkhalifah, 1997)

$$S = 1 + 8\eta. \quad (37)$$

The difference between equations (36) and (37) is an additional indicator of the fundamental difference between homogeneous VTI and vertically heterogeneous isotropic media. The three-parameter anisotropic approximation (12) can match the reflection moveout in the isotropic model up to the fourth-order term in the Taylor series expansion if the value of η is chosen in accordance with equation (37). We can estimate the error of such an approximation with an equation analogous to (32):

$$\frac{\Delta t^2(l)}{t^2(0)} = \frac{1}{8}(S_3 - 2 + 3S_2 - 2S_2^2) \left(\frac{l}{t_0 V_n} \right)^6. \quad (38)$$

The difference between the error estimates (32) and (38) is

$$\frac{\Delta t^2(l)}{t^2(0)} = \frac{1}{8}(2 - S_2)(S_2 - 1) \left(\frac{l}{t_0 V_n} \right)^6. \quad (39)$$

For usual values of S_2 , which range from 1 to 2, the expression (39) is positive. This means that the anisotropic approximation (12) overestimates traveltimes in the isotropic heterogeneous model even more than does the shifted hyperbola (30) shown in Figure 3b. Below, we examine which of the two approximations is more suitable when the model includes both vertical heterogeneity and anisotropy.

Vertically heterogeneous VTI model

In a model that includes vertical heterogeneity and anisotropy, both factors influence bending of the rays. The weak anisotropy approximation, however, allows us to neglect the effect of anisotropy on ray trajectories and consider its influence on traveltimes only. This assumption is analogous to the linearization, conventionally done for tomographic inversion. Its application to weak anisotropy has been discussed by Grechka and McMechan (1996). According to the linearization assumption, we can retain isotropic equation (20) describing the ray trajectories and rewrite equation (19) in the form

$$t(p) = 2 \int_0^z \frac{dz}{V_g(z, \psi(z)) \cos \psi(z)}, \quad (40)$$

where V_g is the anisotropic group velocity, which varies both with the depth z and with the ray angle ψ and has the expression (1). Differentiation of the parametric travel-time equations (40) and (20) and linearization with respect to Thomsen's anisotropic parameters shows that the general form of equations (23)–(26) remains valid if we replace the definitions of the root-mean-square velocity V_{rms} and the parameters M_k by

$$V_{rms}^2 = \frac{1}{t_z} \int_0^{t_z} V_z^2(t) [1 + 2\delta(t)] dt, \quad (41)$$

$$M_k = \frac{1}{t_z} \int_0^{t_z} V_z^{2k}(t) [1 + 2\delta(t)]^{2k} [1 + 8\eta(t)] dt \quad (k = 2, 3, \dots). \quad (42)$$

In homogeneous media, expressions (41) and (42) transform series (22) with coefficients (23)–(26) into the form equivalent to series (13). Two important conclusions follow from equations (41) and (42). First, if the mean value of the anisotropic coefficient δ is less than zero, the presence of anisotropy can reduce the difference between the effective root-mean-square velocity and the effective vertical velocity $\widehat{V}_z = z/t_z$. In this case, the influence of anisotropy and heterogeneity partially cancel each other, and the moveout curve may behave at small offsets as if the medium were homogeneous and isotropic. This behavior has been noticed by Larner and Cohen (1993). On the other hand, if the anellipticity coefficient η is positive and different from zero, it can significantly increase the values of the heterogeneity parameters S_k defined by equations (29). Then, the nonhyperbolicity of reflection moveouts at large offsets is stronger than that in isotropic media.

To exemplify the general theory, let us consider a simple analytic model with constant anisotropic parameters and the vertical velocity linearly increasing with depth according to the equation

$$V_z(z) = V_z(0) (1 + \beta z) = V_z(0) e^{\kappa(z)}, \quad (43)$$

where κ is the logarithm of the velocity change. In this case, the analytic expression for the RMS velocity V_{rms} is found from equation (41) to be

$$V_{rms}^2 = V_z^2(0) (1 + 2\delta) \frac{e^{2\kappa} - 1}{2\kappa}, \quad (44)$$

while the mean vertical velocity is

$$\widehat{V}_z = \frac{z}{t_z} = V_z(0) \frac{e^\kappa - 1}{\kappa}, \quad (45)$$

where $\kappa = \kappa(z)$ is evaluated at the reflector depth. Comparing equations (44) and (45), we can see that the squared RMS velocity V_{rms}^2 equals to the squared mean velocity \widehat{V}_z^2 if

$$1 + 2\delta = \frac{2(e^\kappa - 1)}{\kappa(e^\kappa + 1)}. \quad (46)$$

For small κ , the estimate of δ from equation (46) is

$$\delta \approx -\frac{\kappa^2}{24}. \quad (47)$$

For example, if the vertical velocity near the reflector is twice that at the surface (i.e., $\kappa = \ln 2 \approx 0.69$), having the anisotropic parameter δ as small as -0.02 is sufficient to cancel out the influence of heterogeneity on the normal-moveout velocity. The values of parameters S_2 and S_3 , found from equations (29), (41) and (42), are

$$S_2 = (1 + 8\eta) \kappa \frac{e^{2\kappa} + 1}{e^{2\kappa} - 1}, \quad (48)$$

$$S_3 = \frac{4}{3} (1 + 8\eta) \kappa^2 \frac{e^{4\kappa} + e^{2\kappa} + 1}{(e^{2\kappa} - 1)^2}. \quad (49)$$

Substituting equations (48) and (49) into the estimates (32) and (38) and linearizing them both in η and in κ , we find that the error of anisotropic traveltimes approximation (12) in the linear velocity model is

$$\frac{\Delta t^2(l)}{t^2(0)} = -\frac{\kappa^2(1 - 8\eta)}{12} \left(\frac{l}{t_0 V_n} \right)^6, \quad (50)$$

while the error of the shifted-hyperbola approximation (30) is

$$\frac{\Delta t^2(l)}{t^2(0)} = \left(\frac{\kappa^2(1 - 8\eta)}{24} - \eta \right) \left(\frac{l}{t_0 V_n} \right)^6. \quad (51)$$

Comparing equations (50) and (51), we conclude that if the medium is elliptically anisotropic ($\eta = 0$), the shifted hyperbola can be twice as accurate as the anisotropic equation (assuming the optimal choice of parameters). The accuracy of the latter, however, increases when the anellipticity coefficient η grows and becomes higher than that of the shifted hyperbola if η satisfies the approximate inequality

$$\eta \geq \frac{\kappa^2}{8(1 + \kappa^2)}. \quad (52)$$

For instance, if $\kappa = \ln 2$, inequality (52) yields $\eta \geq 0.03$, a quite small value.

CURVILINEAR REFLECTOR

Reflector curvature can also cause nonhyperbolic reflection moveout. In isotropic media, local dip of the reflector influences the normal-moveout velocity, while reflector curvature introduces nonhyperbolic moveout. When overlaying layer is also anisotropic, both hyperbolic and nonhyperbolic moveouts for reflections from curved reflectors also become functions of the anisotropic parameters.

Curved reflector beneath isotropic medium

If the reflector has the shape of a dipping plane beneath a homogeneous isotropic medium, the reflection moveout in the dip direction is a hyperbola (Levin, 1971)

$$t^2(l) = t_0^2 + \frac{l^2}{V_n^2}. \quad (53)$$

Here

$$t_0 = \frac{2L}{V_z}, \quad (54)$$

$$V_n = \frac{V_z}{\cos \alpha}, \quad (55)$$

L is the length of the zero-offset ray, and α is the reflector dip. Formula (53) is inaccurate if the reflector is both dipping and curved. The Taylor series expansion for moveout in this case has the form of equation (22), with coefficients (Fomel, 1994)

$$a_2 = \frac{\cos^2 \alpha \sin^2 \alpha G}{4 V_z^2 L^2}, \quad (56)$$

$$a_3 = -\frac{\cos^2 \alpha \sin^2 \alpha G^2}{16 V_z^2 L^4} \left(\cos 2\alpha + \sin 2\alpha \frac{G K_3}{K_2^2 L} \right), \quad (57)$$

where

$$G = \frac{K_2 L}{1 + K_2 L}, \quad (58)$$

K_2 is the reflector curvature [defined by equation (61)] at the reflection point of the zero-offset ray, and K_3 is the third-order curvature [equation (62)]. If the reflector has an explicit representation $z = z(x)$, then the parameters in equations (56) and (57) are

$$\tan \alpha = \frac{dz}{dx}, \quad (59)$$

$$L = \frac{z}{\cos \alpha}, \quad (60)$$

$$K_2 = \frac{d^2 z}{dx^2} \cos^3 \alpha, \quad (61)$$

$$K_3 = \frac{d^3 z}{dx^3} \cos^4 \alpha - 3 K_2^2 \tan \alpha. \quad (62)$$

Keeping only three terms in the Taylor series leads to the approximation

$$t^2(l) = t_0^2 + \frac{l^2}{V_n^2} + \frac{G l^4 \tan^2 \alpha}{V_n^2 (V_n^2 t_0^2 + G l^2)}, \quad (63)$$

where we included the denominator in the third term to ensure that the traveltime behavior at large offsets satisfies the obvious limit

$$\lim_{l \rightarrow \infty} t^2(l) = \frac{l^2}{V_z^2}. \quad (64)$$

As indicated by equation (61), the sign of the curvature K_2 is positive if the reflector is locally convex (i.e., an anticline-type). The sign of K_2 is negative for concave, syncline-type reflectors. Therefore, the coefficient G expressed by equation (58) and, likewise, the nonhyperbolic term in (63) can take both positive and negative values. This means that only for concave reflectors in homogeneous media do nonhyperbolic moveouts resemble those in VTI and vertically heterogeneous media. Convex surfaces produce nonhyperbolic moveout with the opposite sign. Clearly, equation (63) is not accurate for strong negative curvatures $K_2 \approx -1/L$, which cause focusing of the reflected rays and triplications of the reflection traveltimes.

In order to evaluate the accuracy of approximation (63), we can compare it with the exact expression for a point diffractor, which is formally a convex reflector with an infinite curvature. The exact expression for normal moveout in the present notation is

$$t(l) = \frac{\sqrt{z^2 + (z \tan \alpha - l/2)^2} + \sqrt{z^2 + (z \tan \alpha + l/2)^2}}{V_z}, \quad (65)$$

where z is the depth of the diffractor, and α is the angle from vertical of the zero-offset ray. Figure 4 shows the relative error of approximation (63) as a function of the ray angle for offset l twice the diffractor depth z . The maximum error of about 1% occurs at $\alpha \approx 50^\circ$. We can expect equation (63) to be even more accurate for reflectors with smaller curvatures.

Curved reflector beneath homogeneous VTI medium

For a dipping curved reflector in a homogeneous VTI medium, the ray trajectories of the incident and reflected waves are straight, but the location of the reflection point is no longer controlled by the isotropic laws. To obtain analytic expressions in this model, we use the theorem that connects the derivatives of the common-midpoint traveltime with the derivatives of the one-way traveltimes for an imaginary wave originating at the reflection point of the zero-offset ray. This theorem, introduced for the second-order derivatives by Chernjak and Gritsenko (1979), is usually called the normal incidence point (NIP) theorem (Hubral and Krey, 1980; Hubral, 1983). Although the original proof did not address anisotropy, it is applicable to anisotropic media because it is based on the fundamental Fermat's principle. The "normal incidence" point in anisotropic media is the point of incidence for the zero-offset ray

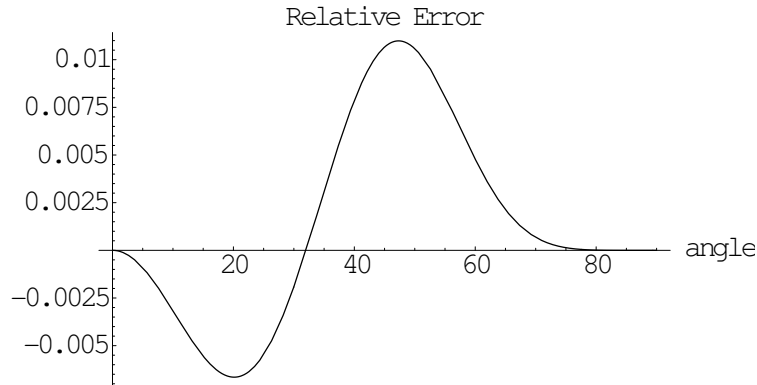


Figure 4: Relative error e of the nonhyperbolic moveout approximation (63) for a point diffractor. The error corresponds to offset l twice the diffractor depth z and is plotted against the angle from vertical α of the zero-offset ray.

(which is, in general, not normal to the reflector). In Appendix A, we review the NIP theorem, as well as its extension to the high-order traveltime derivatives (Fomel, 1994).

Two important equations derived in Appendix A are:

$$\left. \frac{\partial^2 t}{\partial l^2} \right|_{l=0} = \frac{1}{2} \frac{\partial^2 T}{\partial y^2}, \quad (66)$$

$$\left. \frac{\partial^4 t}{\partial l^4} \right|_{l=0} = \frac{1}{8} \frac{\partial^4 T}{\partial y^4} - \frac{3}{8} \left(\frac{\partial^2 T}{\partial x^2} \right)^{-1} \left(\frac{\partial^3 T}{\partial y^2 \partial x} \right)^2, \quad (67)$$

where $T(x, y)$ is the one-way traveltime of the direct wave propagating from the reflection point x to the point y at the surface $z = 0$. All derivatives in equations (66) and (67) are evaluated at the zero-offset ray. Both equations are based solely on Fermat's principle and, therefore, remain valid in any type of media for reflectors of an arbitrary shape, assuming that the traveltimes possess the required order of smoothness. It is especially convenient to use equations (66) and (67) in homogeneous media, where the direct traveltime T can be expressed explicitly.

To apply equations (66) and (67) in VTI media, we need to start with tracing the zero-offset ray. According to Fermat's principle, the ray trajectory must correspond to an extremum of the traveltime. For the zero-offset ray, this simply means that the one-way traveltime T satisfies the equation

$$\frac{\partial T}{\partial x} = 0, \quad (68)$$

where

$$T(x, y) = \frac{\sqrt{z^2(x) + (x - y)^2}}{V_g(\psi(x, y))}. \quad (69)$$

Here, the function $z(x)$ describes the reflector shape, and ψ is the ray angle given by the trigonometric relationship (Figure 5)

$$\cos \psi(x, y) = \frac{z(x)}{\sqrt{z^2(x) + (x - y)^2}} . \quad (70)$$

Substituting approximate equation (5) for the group velocity V_g into equation (69) and linearizing it with respect to the anisotropic parameters δ and η , we can solve equation (68) for y , obtaining

$$y = x + z \tan \alpha (1 + 2\delta + 4\eta \sin^2 \alpha) \quad (71)$$

or, in terms of ψ ,

$$\tan \psi = \tan \alpha (1 + 2\delta + 4\eta \sin^2 \alpha) , \quad (72)$$

where α is the local dip of the reflector at the reflection point x . Equation (72) shows that, in VTI media, the angle ψ of the zero-offset ray differs from the reflector dip α (Figure 5). As one might expect, the relative difference is approximately linear in Thomsen anisotropic parameters.

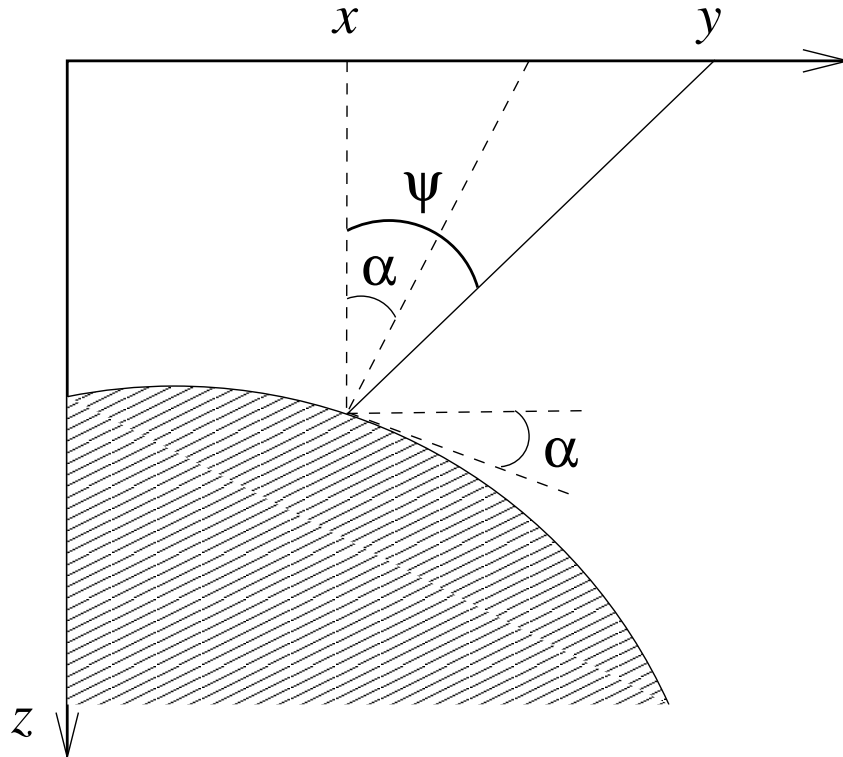


Figure 5: Zero-offset reflection from a curved reflector beneath a VTI medium (a scheme). Note that the ray angle ψ is not equal to the local reflector dip α .

Now we can apply equation (66) to evaluate the second term of the Taylor series expansion (22) for a curved reflector. The linearization in anisotropic parameters

leads to the expression

$$a_1 = \frac{1}{V_n^2} = \frac{\cos^2 \alpha}{V_z^2 (1 + 2\delta(1 + \sin^2 \alpha) + 6\eta \sin^2 \alpha (1 + \cos^2 \alpha))}, \quad (73)$$

which is equivalent to that derived by Tsvankin (1995). As in isotropic media, the normal-moveout velocity does not depend on the reflector curvature. Its dip dependence, however, is an important indicator of anisotropy, especially in areas of conflicting dips (Alkhalifah and Tsvankin, 1995).

Finally, using equation (67), we determine the third coefficient of the Taylor series. After linearization in anisotropic parameters and lengthy algebra, the result takes the form

$$a_2 = \frac{A}{V_n^4 t_0^2}, \quad (74)$$

where

$$A = G \tan^2 \alpha + 2\delta G \sin^2 \alpha (2 + \tan^2 \alpha - G) - 2\eta (1 - 4 \sin^2 \alpha) + 4\eta G \sin^2 \alpha (6 \cos^2 \alpha + \sin^2 \alpha (\tan^2 \alpha - 3G)), \quad (75)$$

and the coefficient G is defined by equation (58). For zero curvature (a plane reflector) $G = 0$, and the only term remaining in equation (75) is

$$A = -2\eta (1 - 4 \sin^2 \alpha). \quad (76)$$

If the reflector is curved, we can rewrite the isotropic equation (63) in the form

$$t^2(l) = t_0^2 + \frac{l^2}{V_n^2} + \frac{A l^4}{V_n^2 (V_n^2 t_0^2 + G l^2)}, \quad (77)$$

where the normal-moveout velocity V_n and the quantity A are given by equations (73) and (75), respectively. Equation (77) approximates the nonhyperbolic moveout in homogeneous VTI media above a curved reflector. For small curvature, the accuracy of this equation at finite offsets can be increased by modifying the denominator in the quartic term similarly to that done by Grechka and Tsvankin (1998) for VTI media.

ANISOTROPY VERSUS LATERAL HETEROGENEITY

The nonhyperbolic moveout in homogeneous VTI media with one horizontal reflector is similar to that caused by lateral heterogeneity in isotropic models. In this section, we discuss this similarity following the results of Grechka (1998).

The angle dependence of the group velocity in equations (1) and (5) is characterized by *small* anisotropic coefficients. Therefore, we can assume that an analogous influence of lateral heterogeneity might be caused by *small* velocity perturbations.

(Large lateral velocity changes can cause behavior too complicated for analytic description.) An appropriate model is a plane laterally heterogeneous layer with the velocity

$$V(y) = V_0 [1 + c(y)] , \quad (78)$$

where $|c(y)| \ll 1$ is a dimensionless function. The velocity $V(y)$ given by equation (78) has the generic perturbation form that allows us to use the tomographic linearization assumption. That is, we neglect the ray bending caused by the small velocity perturbation c and compute the perturbation of traveltimes along straight rays in the constant-velocity background. Thus, we can rewrite equation (9) as

$$t(l) = \frac{\sqrt{4z^2 + l^2}}{l} \int_{y-l/2}^{y+l/2} \frac{d\xi}{V_z(\xi)} , \quad (79)$$

where y is the midpoint location and the integration limits correspond to the source and receiver locations. For simplicity and without loss of generality, we can set y to zero. Linearizing equation (79) with respect to the small perturbation $c(y)$, we get

$$t(l) = \frac{\sqrt{4z^2 + l^2}}{V_0} \left[1 - \frac{1}{l} \int_{-l/2}^{l/2} c(\xi) d\xi \right] . \quad (80)$$

It is clear from equation (80) that lateral heterogeneity can cause many different types of the nonhyperbolic moveout. In particular, comparing equations (80) and (11), we conclude that a pseudo-anisotropic behavior of traveltimes is produced by lateral heterogeneity in the form

$$c(l) = \frac{d}{dl} \left[\frac{l^3(l^2\epsilon + 4z^2\delta)}{(l^2 + 4z^2)^2} \right] \quad (81)$$

or, in the linear approximation,

$$c(l) = \frac{4\delta t_0^2 V_n^2 l^2 (3t_0^2 V_n^2 - l^2) + \epsilon l^4 (5t_0^2 V_n^2 + l^2)}{16(t_0^2 V_n^2 + l^2)^3} , \quad (82)$$

where δ and ϵ should be considered now as parameters, describing the *isotropic* laterally heterogeneous velocity field. Equation (82) indicates that the velocity heterogeneity $c(y)$ that reproduces moveout (12) in a homogeneous VTI medium, is a symmetric function of the offset l . This is not surprising because the velocity function (1), corresponding to vertical transverse isotropy, is symmetric as well.

CONCLUSIONS

Nonhyperbolic reflection moveout of P -waves is sometimes considered as an important indicator of anisotropy. Its correct interpretation, however, is impossible without

taking other factors into account. In this paper, we have considered three other important factors: vertical heterogeneity, curvature of the reflector, and lateral heterogeneity. Each of them can have an influence on nonhyperbolic behavior of the reflection moveout comparable to that of anisotropy. In particular, vertical heterogeneity produces a depth-variant anisotropic pattern that differs from that in VTI media. For isotropic media, this pattern is reasonably well approximated by the shifted hyperbola. In a vertically heterogeneous VTI medium, the parameters of anisotropy should be replaced with their effective values. For a curved reflector in a homogeneous VTI medium, we have developed an approximation based on the Taylor series expansion of the traveltimes with both the reflector curvature and the anisotropic parameters entering the nonhyperbolic term. Lateral heterogeneity can effectively mimic the influence of virtually any anisotropy.

The theoretical results of this paper are directly applicable to modeling of the nonhyperbolic moveout. The general formulas connecting the derivatives of reflection traveltimes with those of direct waves are particularly attractive in this context. For smooth velocity models, these formulas reduce the problem of tracing a family of reflected rays to tracing only one zero-offset ray. Practical estimation and inversion of nonhyperbolic moveout is a different and more difficult problem than is the forward one. Given that a variety of reasons might cause similar nonhyperbolic moveout of P -waves, its inversion will be nonunique. Nevertheless, the theoretical guidelines provided by the analytical theory are helpful for the correct formulation of the inverse problems. They explicitly show us which medium parameters we may hope to extract from the kinematics of long-spread P -wave reflection data.

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APPENDIX A

NORMAL MOVEOUT BEYOND THE NIP THEOREM

In this Appendix, we derive equations that relate traveltimes derivatives of the reflected wave, evaluated at the zero offset point, and traveltimes derivatives of the direct wave, evaluated in the vicinity of the zero-offset ray. Such a relationship for second-order derivatives is known as the NIP (normal incidence point) theorem (Chernjak and

Gritsenko, 1979; Hubral and Krey, 1980; Hubral, 1983). Its extension to high-order derivatives is described by Fomel (1994).

Reflection traveltimes in any type of model can be considered as a function of the source and receiver locations s and r and the location of the reflection point x , as follows:

$$t(y, h) = F(y, h, x(y, h)) , \quad (\text{A-1})$$

where y is the midpoint ($y = \frac{s+r}{2}$), h is the half-offset ($h = \frac{r-s}{2}$), and the function F has a natural decomposition into two parts corresponding to the incident and reflected rays:

$$F(y, h, x) = T(y - h, x) + T(y + h, x) , \quad (\text{A-2})$$

where T is the traveltimes of the direct wave. Clearly, at the zero-offset point,

$$t(y, 0) = 2T(y, x) , \quad (\text{A-3})$$

where $x = x(y, 0)$ corresponds to the reflection point of the zero-offset ray.

Differentiating equation (A-1) with respect to the half-offset h and applying the chain rule, we obtain

$$\frac{\partial t}{\partial h} = \frac{\partial F}{\partial h} + \frac{\partial F}{\partial x} \frac{\partial x}{\partial h} . \quad (\text{A-4})$$

According to Fermat's principle, one of the fundamental principles of ray theory, the ray trajectory of the reflected wave corresponds to an extremum value of the traveltimes. Parameterizing the trajectory in terms of the reflection point location x and assuming that F is a smooth function of x , we can write Fermat's principle in the form

$$\frac{\partial F}{\partial x} = 0 . \quad (\text{A-5})$$

Equation (A-5) must be satisfied for any values of x and h . Substituting this equation into equation (A-4) leads to the equation

$$\frac{\partial t}{\partial h} = \frac{\partial F}{\partial h} . \quad (\text{A-6})$$

Differentiating (A-6) again with respect to h , we arrive at the equation

$$\frac{\partial^2 t}{\partial h^2} = \frac{\partial^2 F}{\partial h^2} + \frac{\partial^2 F}{\partial h \partial x} \frac{\partial x}{\partial h} . \quad (\text{A-7})$$

Interchanging the source and receiver locations doesn't change the reflection point position (the principle of reciprocity). Therefore, x is an even function of the offset h , and we can simplify equation (A-7) at zero offset, as follows:

$$\left. \frac{\partial^2 t}{\partial h^2} \right|_{h=0} = \left. \frac{\partial^2 F}{\partial h^2} \right|_{h=0} . \quad (\text{A-8})$$

Substituting the expression for the function F (A-2) into (A-8) leads to the equation

$$\left. \frac{\partial^2 t}{\partial h^2} \right|_{h=0} = 2 \frac{\partial^2 T}{\partial y^2}, \quad (\text{A-9})$$

which is the mathematical formulation of the NIP theorem. It proves that the second-order derivative of the reflection traveltime with respect to the offset is equal, at zero offset, to the second derivative of the direct wave traveltime for the wave propagating from the incidence point of the zero-offset ray. One immediate conclusion from the NIP theorem is that the short-spread normal moveout velocity, connected with the derivative in the left-hand-side of equation (A-9) can depend on the reflector dip but doesn't depend on the curvature of the reflector. Our derivation up to this point has followed the derivation suggested by Chernjak and Gritsenko (1979).

Differentiating equation (A-7) twice with respect to h evaluates, with the help of the chain rule, the fourth-order derivative, as follows:

$$\frac{\partial^4 t}{\partial h^4} = \frac{\partial^4 F}{\partial h^4} + 3 \frac{\partial^4 F}{\partial h^3 \partial x} \frac{\partial x}{\partial h} + 3 \frac{\partial^4 F}{\partial h^2 \partial x^2} \left(\frac{\partial x}{\partial h} \right)^2 + 3 \frac{\partial^4 F}{\partial h \partial x^3} \left(\frac{\partial x}{\partial h} \right)^3 + \quad (\text{A-10})$$

$$+ 3 \frac{\partial^3 F}{\partial h^2 \partial x} \frac{\partial^2 x}{\partial h^2} + 3 \frac{\partial^3 F}{\partial h \partial x^2} \frac{\partial^2 x}{\partial h^2} \frac{\partial x}{\partial h} + \frac{\partial^2 F}{\partial h \partial x} \frac{\partial^3 x}{\partial h^3}. \quad (\text{A-11})$$

Again, we can apply the principle of reciprocity to eliminate the odd-order derivatives of x in equation (A-11) at the zero offset. The resultant expression has the form

$$\left. \frac{\partial^4 t}{\partial h^4} \right|_{h=0} = \left(\frac{\partial^4 F}{\partial h^4} + 3 \frac{\partial^3 F}{\partial h^2 \partial x} \frac{\partial^2 x}{\partial h^2} \right) \Big|_{h=0}. \quad (\text{A-12})$$

In order to determine the unknown second derivative of the reflection point location $\frac{\partial^2 x}{\partial h^2}$, we differentiate Fermat's equation (A-5) twice, obtaining

$$\frac{\partial^3 F}{\partial^2 h \partial x} + 2 \frac{\partial^3 F}{\partial h \partial x} \frac{\partial x}{\partial h} + \frac{\partial^3 F}{\partial^3 x} \left(\frac{\partial x}{\partial h} \right)^2 + \frac{\partial^2 F}{\partial^2 x} \frac{\partial^2 x}{\partial h^2} = 0. \quad (\text{A-13})$$

Simplifying this equation at zero offset, we can solve it for the second derivative of x . The solution has the form

$$\left. \frac{\partial^2 x}{\partial h^2} \right|_{h=0} = - \left[\left(\frac{\partial^2 F}{\partial^2 x} \right)^{-1} \frac{\partial^3 F}{\partial^2 h \partial x} \right]_{h=0}. \quad (\text{A-14})$$

Here we neglect the case of $\frac{\partial^2 F}{\partial^2 x} = 0$, which corresponds to a focusing of the reflected rays at the surface. Finally, substituting expression (A-14) into (A-12) and recalling the definition of the F function from (A-2), we obtain the equation

$$\left. \frac{\partial^4 t}{\partial h^4} \right|_{h=0} = 2 \frac{\partial^4 T}{\partial y^4} - 6 \left(\frac{\partial^2 T}{\partial x^2} \right)^{-1} \left(\frac{\partial^3 T}{\partial y^2 \partial x} \right)^2, \quad (\text{A-15})$$

which is the same as equation (67) in the main text. Higher-order derivatives can be expressed in an analogous way with a set of recursive algebraic functions (Fomel, 1994).

In the derivation of equations (A-9) and (A-15), we have used Fermat's principle, the principle of reciprocity, and the rules of calculus. Both these equations remain valid in anisotropic media as well as in heterogeneous media, providing that the travelttime function is smooth and that focusing of the reflected rays doesn't occur at the surface of observation.

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