Local seismic attributes[a]

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ABSTRACT

Local seismic attributes measure seismic signal characteristics not instantaneously at each signal point and not globally across a data window but locally in the neighborhood of each point. I define local attributes with the help of regularized inversion and demonstrate their usefulness for measuring local frequencies of seismic signals and local similarity between different datasets. I use shaping regularization for controlling the locality and smoothness of local attributes. A multicomponent image registration example from a nine-component land survey illustrates practical applications of local attributes for measuring differences between registered images.

INTRODUCTION

Seismic attribute is defined by Sheriff (1991) as a "measurement derived from seismic data". Such a broad definition allows for many uses and abuses of the term. Countless attributes have been introduced in the practice of seismic exploration, (Brown, 1996; Chen and Sidney, 1997), which led Eastwood (2002) to talk about "attribute explosion". Many of these attributes play an exceptionally important role in interpreting and analyzing seismic data (Chopra and Marfurt, 2005).

In this paper, I consider two particular attribute applications:

1. *Measuring local frequency content in a seismic image* is important both for studying the phenomenon of seismic wave attenuation and for processing of attenuated signals.

2. *Measuring local similarity between two seismic images* is useful for seismic monitoring, registration of multicomponent data, and analysis of velocities and amplitudes.

Some of the best known seismic attributes are instantaneous attributes such as instantaneous phase or instantaneous dip (Taner et al., 1979; Barnes, 1992, 1993). Such attributes measure seismic frequency characteristics as being attached instantaneously to each signal point. This measure is notoriously noisy and may lead to unphysical values such as negative frequencies (White, 1991).

In this paper, I introduce a concept of *local attributes*. Local attributes measure signal characteristics not instantaneously at each data point but in a local neighborhood around the point. According to the Fourier uncertainty principle, frequency is essentially an uncertain characteristic when applied to a local region in the time domain. Therefore, local frequency is more physically meaningful than instantaneous frequency. The idea of locality extends from local frequency to other attributes, such as the correlation coefficient between two different datasets, that are conventionally evaluated in sliding windows.

The paper starts with reviewing the definition of instantaneous frequency. I modify this definition to that of local frequency by recognizing it as a form of regularized inversion and by changing regularization to constrain the continuity and smoothness of the output. The same idea is extended next to define local correlation . I illustrate a practical application of local attributes using an example from multicomponent seismic image registration in a nine-component land survey.

MEASURING LOCAL FREQUENCIES

Definition of instantaneous frequency

Let $f(t)$ represent seismic trace as a function of time t. The corresponding complex trace $c(t)$ is defined as

$$c(t) = f(t) + i\,h(t)\;, \tag{1}$$

where $h(t)$ is the Hilbert transform of the real trace $f(t)$. One can also represent the complex trace in terms of the envelope $A(t)$ and the instantaneous phase $\phi(t)$, as follows:

$$c(t) = A(t)\,e^{i\,\phi(t)}\;. \tag{2}$$

By definition, instantaneous frequency is the time derivative of the instantaneous phase (Taner et al., 1979)

$$\omega(t) = \phi'(t) = Im\left[\frac{c'(t)}{c(t)}\right] = \frac{f(t)\,h'(t) - f'(t)\,h(t)}{f^2(t) + h^2(t)}\;. \tag{3}$$

Different numerical realizations of equation 3 produce slightly different algorithms (Barnes, 1992).

Note that the definition of instantaneous frequency calls for division of two signals. In a linear algebra notation,

$$\mathbf{w} = \mathbf{D}^{-1}\,\mathbf{n}\;, \tag{4}$$

where $\mathbf{w}$ represents the vector of instantaneous frequencies $\omega(t)$, $\mathbf{n}$ represents the numerator in equation 3, and $\mathbf{D}$ is a diagonal operator made from the denominator of equation 3. A recipe for avoiding division by zero is adding a small constant ϵ to the denominator (Matheney and Nowack, 1995). Consequently, equation 4 transforms to

$$\mathbf{w}_{inst} = \left(\mathbf{D} + \epsilon\,\mathbf{I}\right)^{-1}\,\mathbf{n}\;, \tag{5}$$

where **I** stands for the identity operator. Stabilization by ϵ does not, however, prevent instantaneous frequency from being a noisy and unstable attribute. The main reason for that is the extreme locality of the instantaneous frequency measurement, governed only by the phase shift between the signal and its Hilbert transform.

Figure **??** shows three test signals for comparing frequency attributes. The first signal is a synthetic chirp function with linearly varying frequency. Instantaneous frequency shown in Figure 1 correctly estimates the modeled frequency trend. The second signal is a piece of a synthetic seismic trace obtained by convolving a 40-Hz Ricker wavelet with synthetic reflectivity. The instantaneous frequency (Figure 1b) shows many variations and appears to contain detailed information. However, this information is useless for characterizing the dominant frequency content of the data, which remains unchanged due to stationarity of the seismic wavelet. The last test example (Figure **??**c) is a real trace extracted from a seismic image. The instantaneous frequency (Figure 1c) appears noisy and even contains physically unreasonable negative values. Similar behavior was described by White (1991).

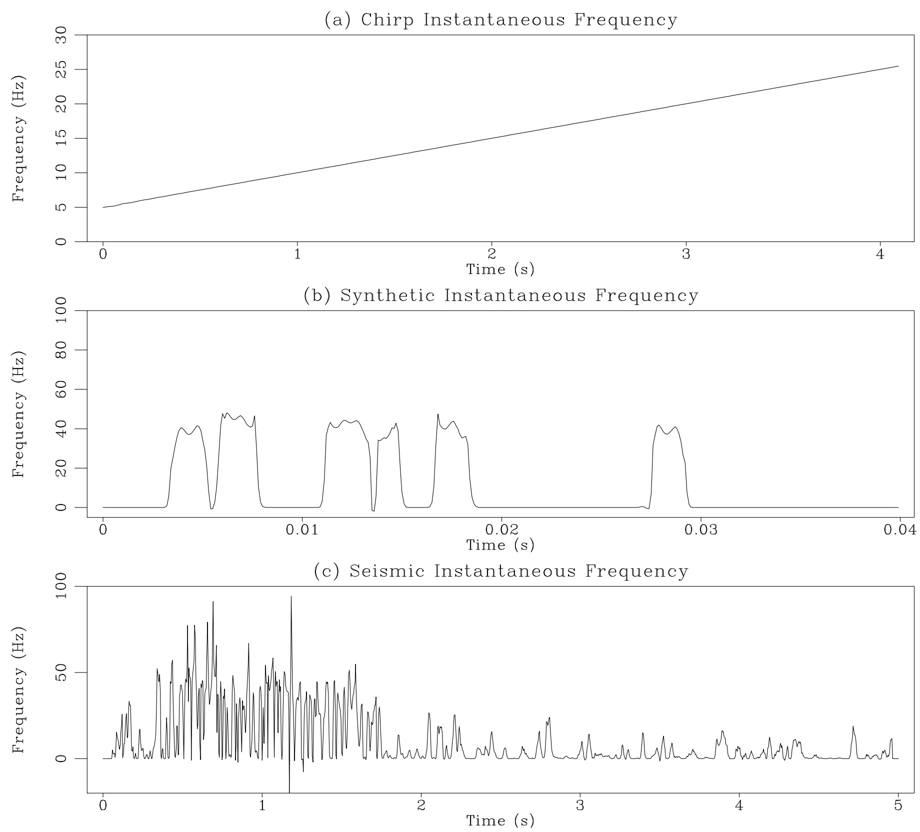

Figure 1: Instantaneous frequency of test signals from Figure **??**.

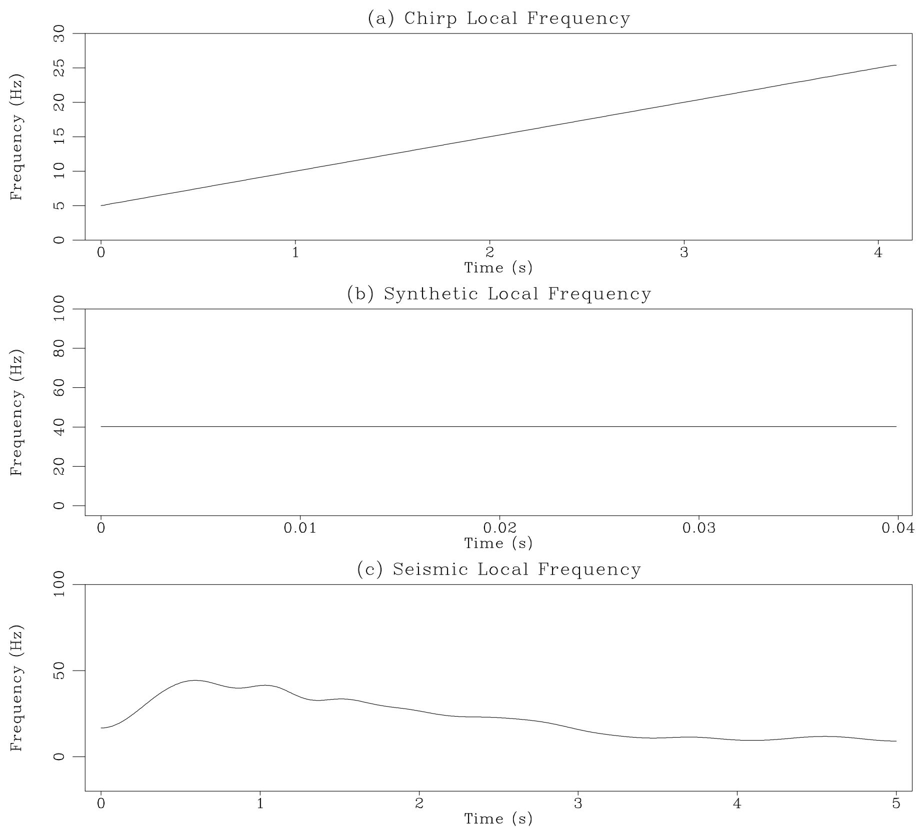

Figure 2: Local frequency of test signals from Figure **??**.

Definition of local frequency

The definition of the local frequency attribute starts by recognizing equation 5 as a regularized form of linear inversion. Changing regularization from simple identity to a more general regularization operator **R** provides the definition for local frequency as follows:

$$\mathbf{w}_{loc} = \left(\mathbf{D} + \epsilon\,\mathbf{R}\right)^{-1}\,\mathbf{n}\;, \tag{6}$$

The role of the regularization operator is ensuring continuity and smoothness of the local frequency measure. A different approach to regularization follows from the shaping method (Fomel, 2006). Shaping regularization operates with a smoothing (shaping) operator **S** by incorporating it into the inversion scheme as follows:

$$\mathbf{w}_{loc} = \left[\lambda^2\,\mathbf{I} + \mathbf{S}\,\left(\mathbf{D} - \lambda^2\,\mathbf{I}\right)\right]^{-1}\,\mathbf{S}\,\mathbf{n}\;, \tag{7}$$

Scaling by λ preserves physical dimensionality and enables fast convergence when inversion is implemented by an iterative method. A natural choice for λ is the least-squares norm of **D**.

Figure 2 shows the results of measuring local frequency in the test signals from Figure **??**. I used the shaping regularization formulation 7 with the shaping operator **S** defined as a triangle smoother. The chirp signal frequency (Figure 2a) is correctly recovered. The dominant frequency of the synthetic signal (Figure 2b) is correctly estimated to be stationary at 40 Hz. The local frequency of the real trace (Figure 2c) appears to vary with time according to the general frequency attenuation trend.

This example highlights some advantages of the local attribute method in comparison with the sliding window approach:

- Only one parameter (the smoothing radius) needs to be specified as opposed to several (window size, overlap, and taper) in the windowing approach. The smoothing radius directly reflects the locality of the measurement.
- The local attribute approach continues the measurement smoothly through the regions of absent information such as the zero amplitude regions in the synthetic example, where the signal phase is undefined. This effect is impossible to achieve in the windowing approach unless the window size is always larger than the information gaps in the signal.

Figure **??** shows seismic images from compressional (PP) and shear (SS) reflections obtained by processing a land nine-component survey. Figure **??** shows local frequencies measured in PP and SS images after warping the SS image into PP time. The term "image warping" comes from medical imaging (Wolberg, 1990) and refers, in this case, to squeezing the SS image to PP reflection time to make the two images display in the same coordinate system. We can observe a general decay of frequency with time caused by seismic attenuation. After mapping (squeezing) to PP time, the SS image frequency appears higher in the shallow part of the image because of a relatively low S-wave velocity but lower in the deeper part of the image because of the apparently stronger attenuation of shear waves. A low-frequency anomaly in the PP image might be indicative of gas presence. Identifying and balancing non-stationary frequency variations of multicomponent images is an essential part of the multistep image registration technique (Fomel and Backus, 2003; Fomel et al., 2005).

MEASURING LOCAL SIMILARITY

Consider the task of measuring similarity between two different signals $a(t)$ and $b(t)$. One can define similarity as a global correlation coefficient and then, perhaps, measure it in sliding windows across the signal. The local construction from the previous section suggests approaching this problem in a more elegant way.

Definition of global correlation

Global correlation coefficient between $a(t)$ and $b(t)$ can be defined as the functional

$$\gamma = \frac{< a(t), b(t) >}{\sqrt{< a(t), a(t) > < b(t), b(t) >}} \;, \tag{8}$$

where $< x(t), y(t) >$ denotes the dot product between two signals:

$$< x(t), y(t) >= \int x(t)\, y(t)\, dt \;.$$

According to definition 8, the correlation coefficient of two identical signals is equal to one, and the correlation of two signals with opposite polarity is minus one. In all the other cases, the correlation will be less then one in magnitude thanks to the Cauchy-Schwartz inequality.

The global measure 8 is inconvenient because it supplies only one number for the whole signal. The goal of local analysis is to turn the functional into an operator and to produce local correlation as a variable function $\gamma(t)$ that identifies local changes in the signal similarity.

Definition of local correlation

In a linear algebra notation, the squared correlation coefficient γ from equation 8 can be represented as a product of two least-squares inverses

$$\gamma^2 = \gamma_1\, \gamma_2 \;, \tag{9}$$

$$\gamma_1 = \left(\mathbf{a}^T\, \mathbf{a}\right)^{-1} \left(\mathbf{a}^T\, \mathbf{b}\right) \;, \tag{10}$$

$$\gamma_2 = \left(\mathbf{b}^T\, \mathbf{b}\right)^{-1} \left(\mathbf{b}^T\, \mathbf{a}\right) \;, \tag{11}$$

where $\mathbf{a}$ is a vector notation for $a(t)$, $\mathbf{b}$ is a vector notation for $b(t)$, and $\mathbf{x}^T\,\mathbf{y}$ denotes the dot product operation. Let $\mathbf{A}$ be a diagonal operator composed from the elements of $\mathbf{a}$ and $\mathbf{B}$ be a diagonal operator composed from the elements of $\mathbf{b}$. Localizing equations 10-11 amounts to adding regularization to inversion. Scalars γ_1 and γ_2 turn into vectors $\mathbf{c}_1$ and $\mathbf{c}_2$ defined, using shaping regularization , as

$$\mathbf{c}_1 = \left[\lambda^2\,\mathbf{I} + \mathbf{S}\,\left(\mathbf{A}^T\,\mathbf{A} - \lambda^2\,\mathbf{I}\right)\right]^{-1}\,\mathbf{S}\,\mathbf{A}^T\,\mathbf{b}\,, \tag{12}$$

$$\mathbf{c}_2 = \left[\lambda^2\,\mathbf{I} + \mathbf{S}\,\left(\mathbf{B}^T\,\mathbf{B} - \lambda^2\,\mathbf{I}\right)\right]^{-1}\,\mathbf{S}\,\mathbf{B}^T\,\mathbf{a}\,. \tag{13}$$

To define a local similarity measure, I apply the component-wise product of vectors $\mathbf{c}_1$ and $\mathbf{c}_2$. It is interesting to note that, if one applies an iterative conjugate-gradient inversion for computing the inverse operators in equations 12 and 13, the output of the first iteration will be the smoothed product of the two signals $\mathbf{c}_1 = \mathbf{c}_2 = \mathbf{S}\,\mathbf{A}^T\,\mathbf{b}$, which is equivalent, with an appropriate choice of $\mathbf{S}$, to the algorithm of fast local cross-correlation proposed by Hale (2006).

The local similarity attribute is useful for solving the problem of multicomponent image registration. After an initial registration using interpreter's "nails" (DeAngelo et al., 2004) or velocities from seismic processing, a useful registration indicator is obtained by squeezing and stretching the warped shear-wave image while measuring its local similarity to the compressional image. Such a technique was named *residual γ scan* and proposed by Fomel et al. (2005). Figure **??** shows a residual scan for registration of multicomponent images from Figure **??**. Identifying and picking points of high local similarity enables multicomponent registration with high-resolution accuracy. The registration result is visualized in Figure **??**, which shows interleaved traces from PP and SS images before and after registration. The alignment of main seismic events is an indication of successful registration.

CONCLUSIONS

I have introduced a concept of local seismic attributes and specified it for such attributes as local frequency and local similarity. Local attributes measure signal characteristics not instantaneously at each signal point and not globally across a data window but locally in the neighborhood of each point. They find applications in different steps of multicomponent seismic image registration. One can extend the idea of local attributes to other applications.

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