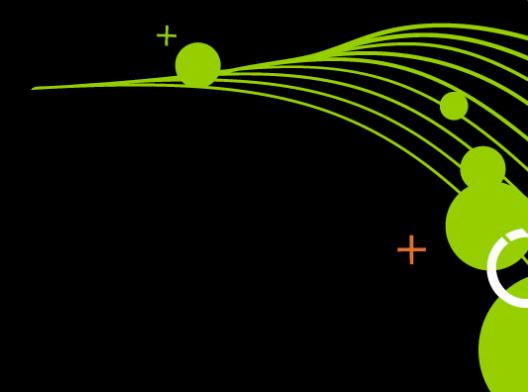




Seismic Interferometry

Madagascar School on Reproducible Computational Geophysics

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GXT Imaging Solutions
Ivan.Vasconcelos@iongeo.com



What we do



- Sensors, sources & navigation
- Seismic services (Land & Marine)
- Data library (e.g., *Spans*)

Topics



- Interferometry of full fields
- Interferometry of scattered fields
- Application examples:
 - Interferometry by deconvolution
 - Dual-field OBC data
 - Imaging internal multiples
- Exercises

Topics



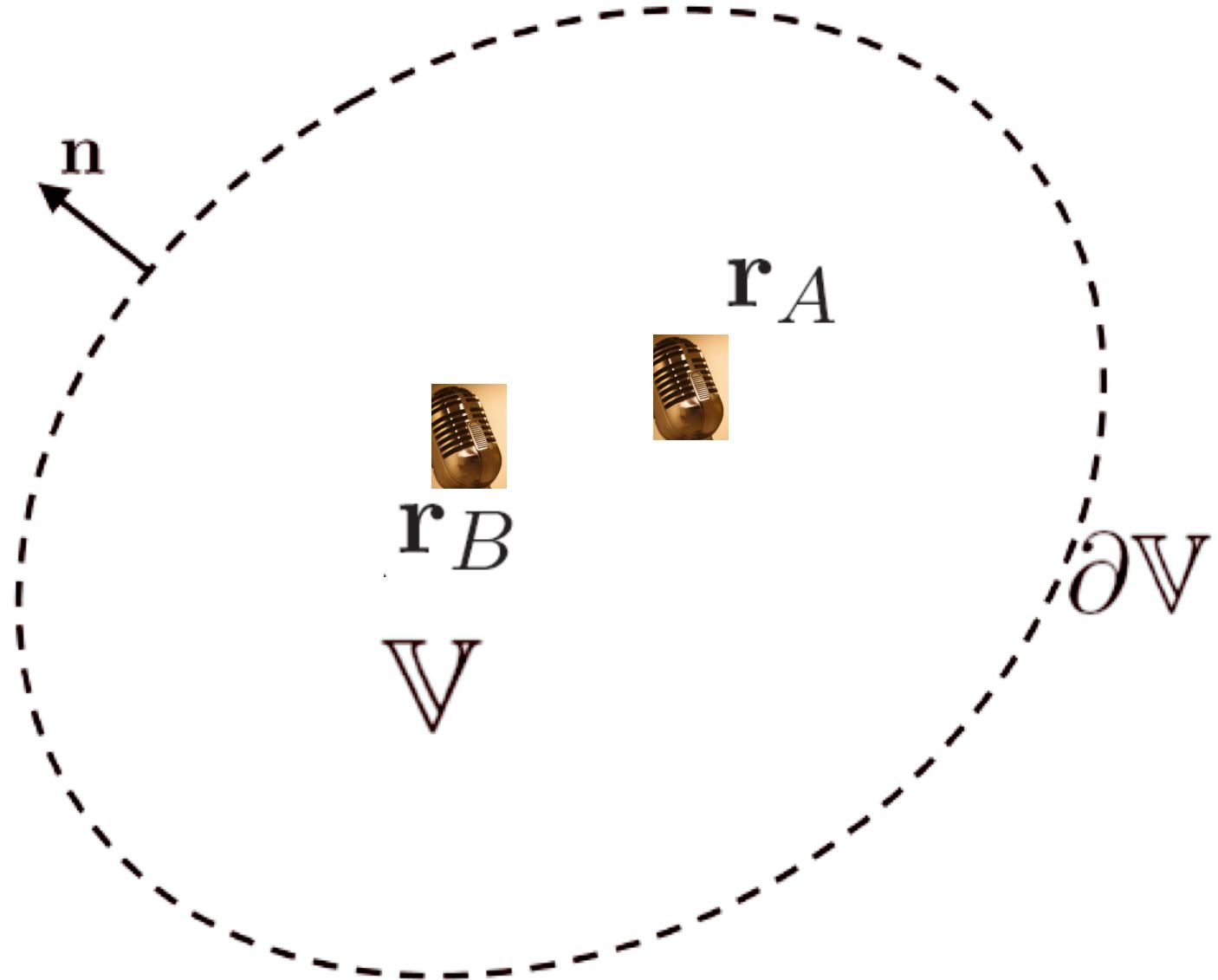
- Interferometry of full fields
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References

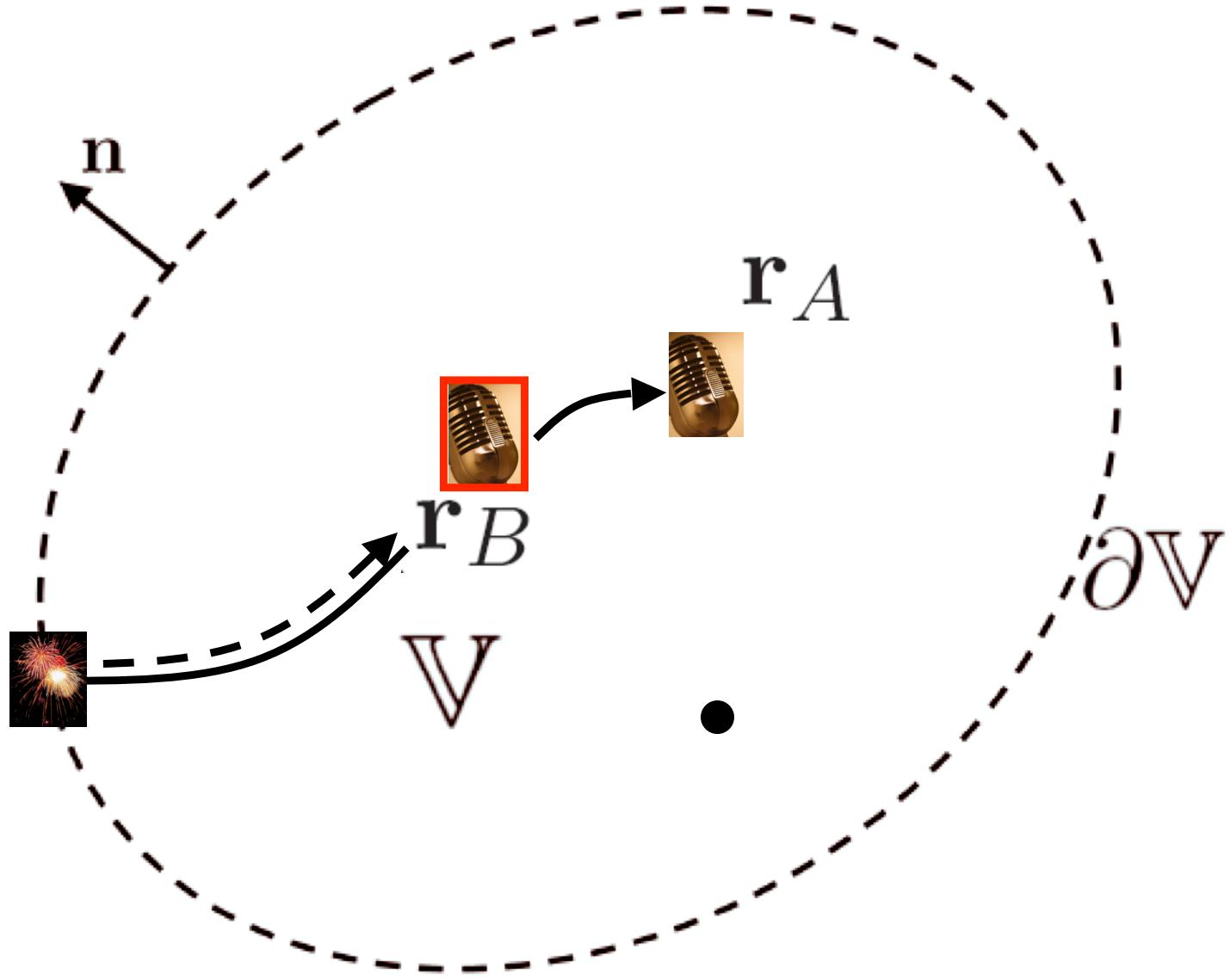


- For these slides
 - Vasconcelos, Snieder & Douma, 2009, Phys. Rev. E, accepted (pre-print available from <http://web.mac.com/ivasconcelos>);
 - Vasconcelos & Snieder, 2008, Geophysics, **73**, S115-S118;
 - Vasconcelos, Snieder & Hornby, 2008, Geophysics, **73**, S157-S168.
- For interferometry:
 - Wapenaar, Slob, Draganov et al.
 - Snieder et al.
 - Schuster et al.
 - Curtis et al.
 - Fink et al.
 - Weaver, Lobkis, Larose et al.
 - ...

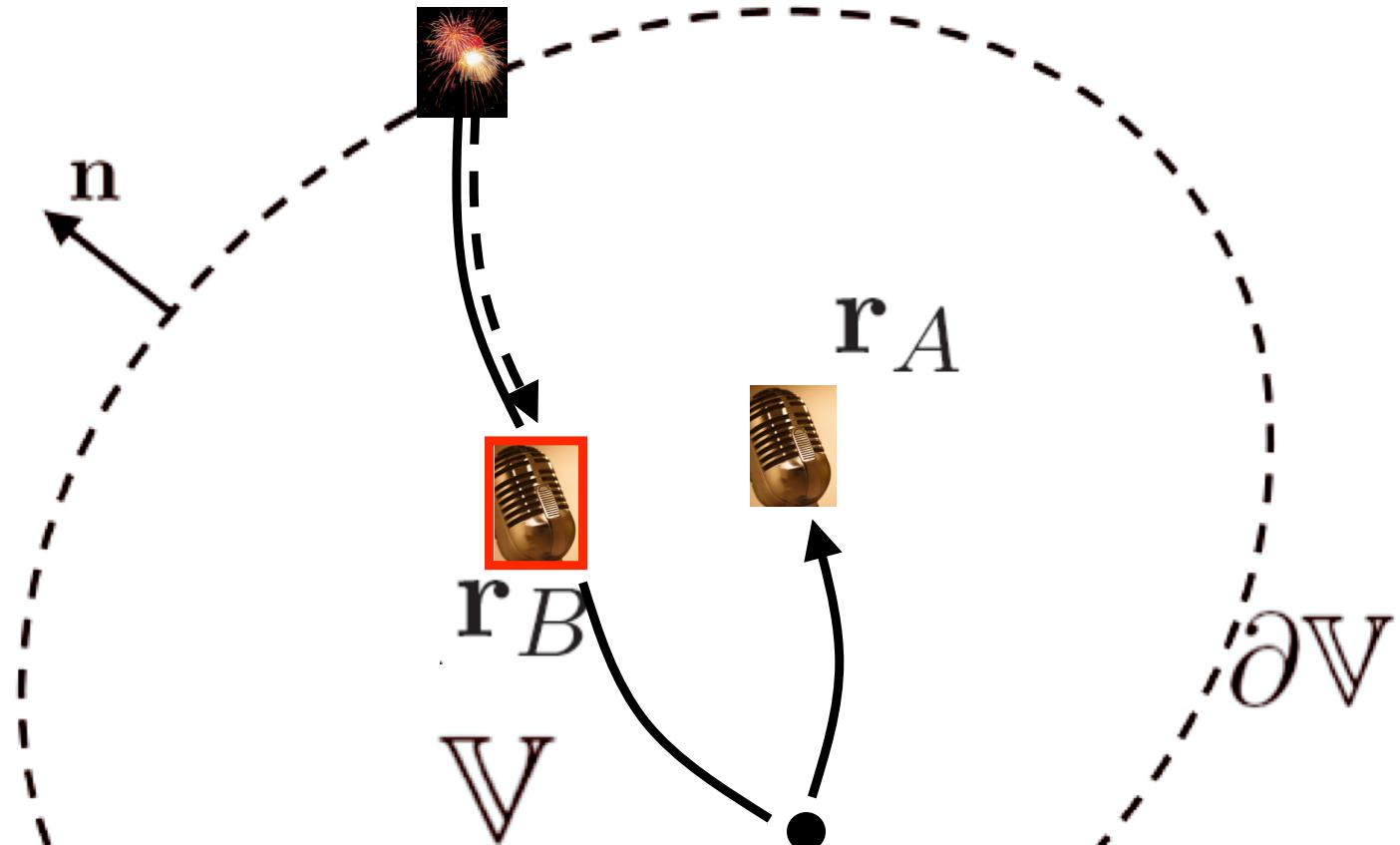
Our experiment configuration



An experiment



A scattered wave



Lossless acoustic waves



$$\nabla p^A(\mathbf{r}, \omega) - i\omega\rho(\mathbf{r})\mathbf{v}^A(\mathbf{r}, \omega) = 0$$

$$\nabla \cdot \mathbf{v}^A(\mathbf{r}, \omega) - i\omega\kappa(\mathbf{r})p^A(\mathbf{r}, \omega) = q^A(\mathbf{r}, \omega)$$

Interaction quantity: $\nabla \cdot (p^A \mathbf{v}^{B*} + p^{B*} \mathbf{v}^A)$

Flux vector

Reciprocity and representation

+

$$\oint_{\mathbf{r} \in \partial \mathbb{V}} [p^A \mathbf{v}^{B*} + p^{B*} \mathbf{v}^A] \cdot d\mathbf{S} = \int_{\mathbf{r} \in \mathbb{V}} [p^A q^{B*} + p^{B*} q^A] dV$$



$$G(\mathbf{r}_A, \mathbf{r}_B) - G^*(\mathbf{r}_A, \mathbf{r}_B) = \oint_{\partial V} \frac{1}{\rho} \left(G^*(\mathbf{r}_B, \mathbf{r}) \frac{\partial G(\mathbf{r}_A, \mathbf{r})}{\partial n} - \frac{\partial G^*(\mathbf{r}_B, \mathbf{r})}{\partial n} G(\mathbf{r}_A, \mathbf{r}) \right) dS.$$

Interferometry - full wavefield

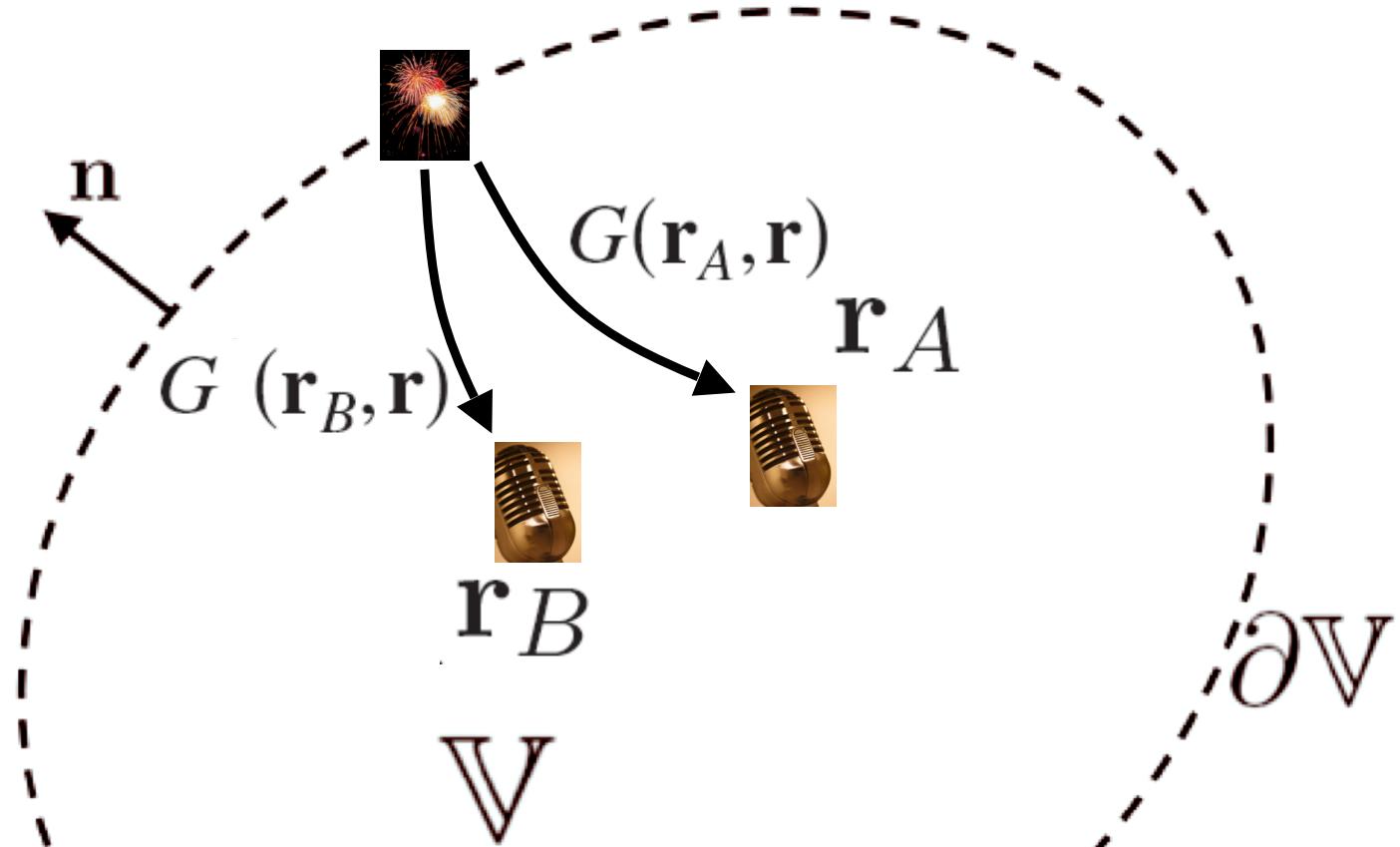


$$G(\mathbf{r}_A, \mathbf{r}_B) - G^*(\mathbf{r}_A, \mathbf{r}_B) = 2i\omega \oint_{\partial V} \frac{1}{\rho c} G(\mathbf{r}_A, \mathbf{r}) G^*(\mathbf{r}_B, \mathbf{r}) dS$$

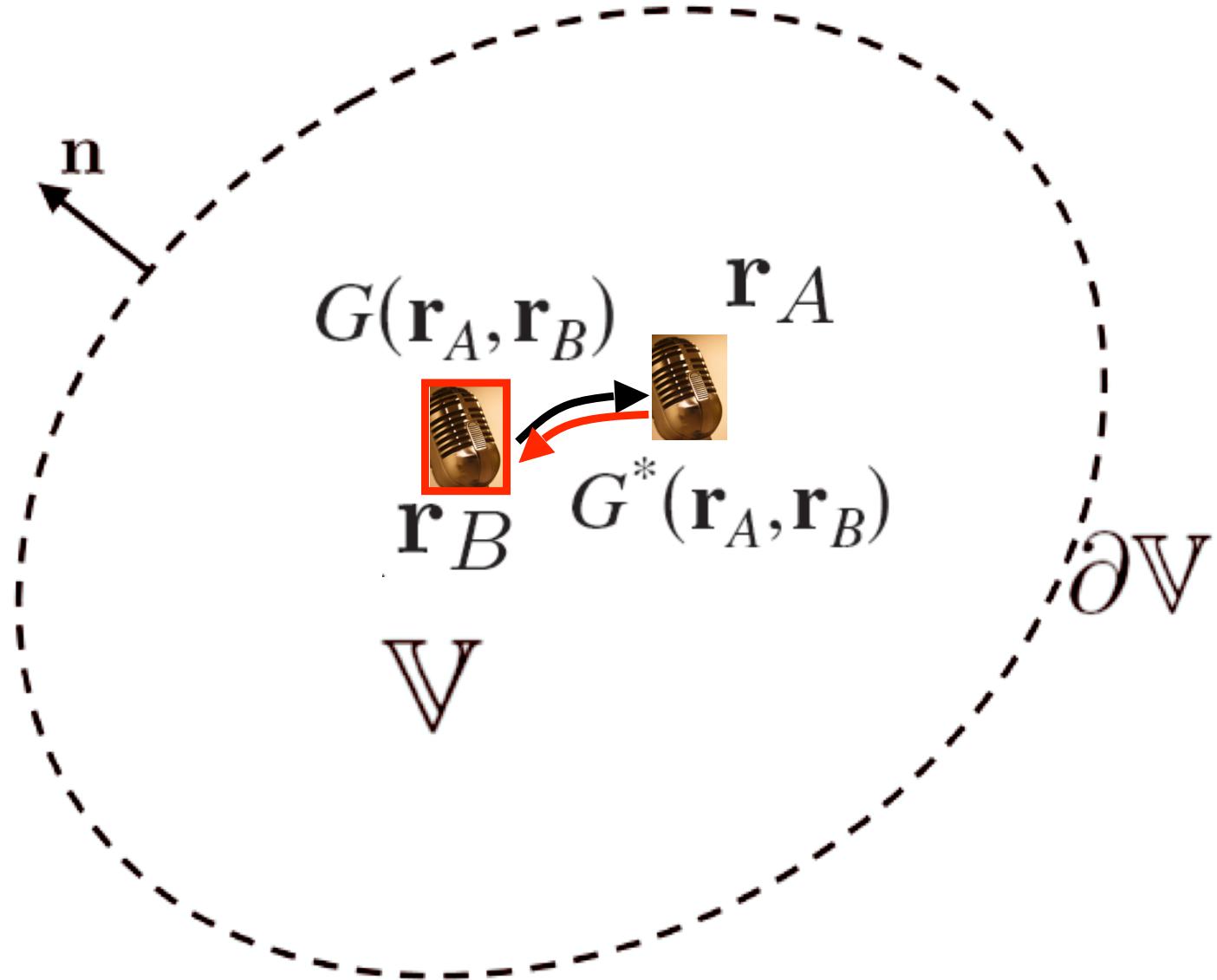
observed

using $\frac{\partial G(\mathbf{r}_0, \mathbf{r})}{\partial n} = ikG(\mathbf{r}_0, \mathbf{r})$

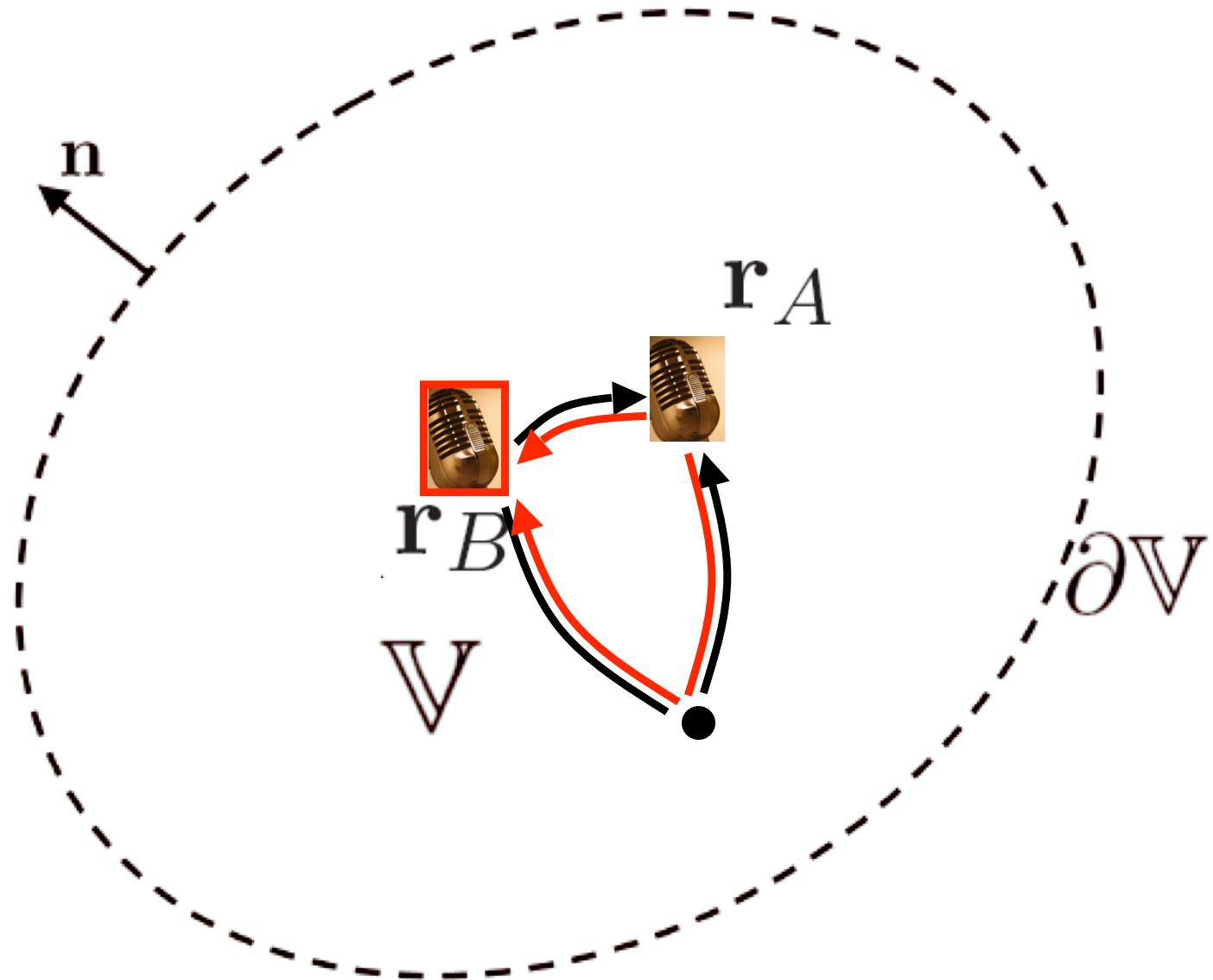
Surface sources



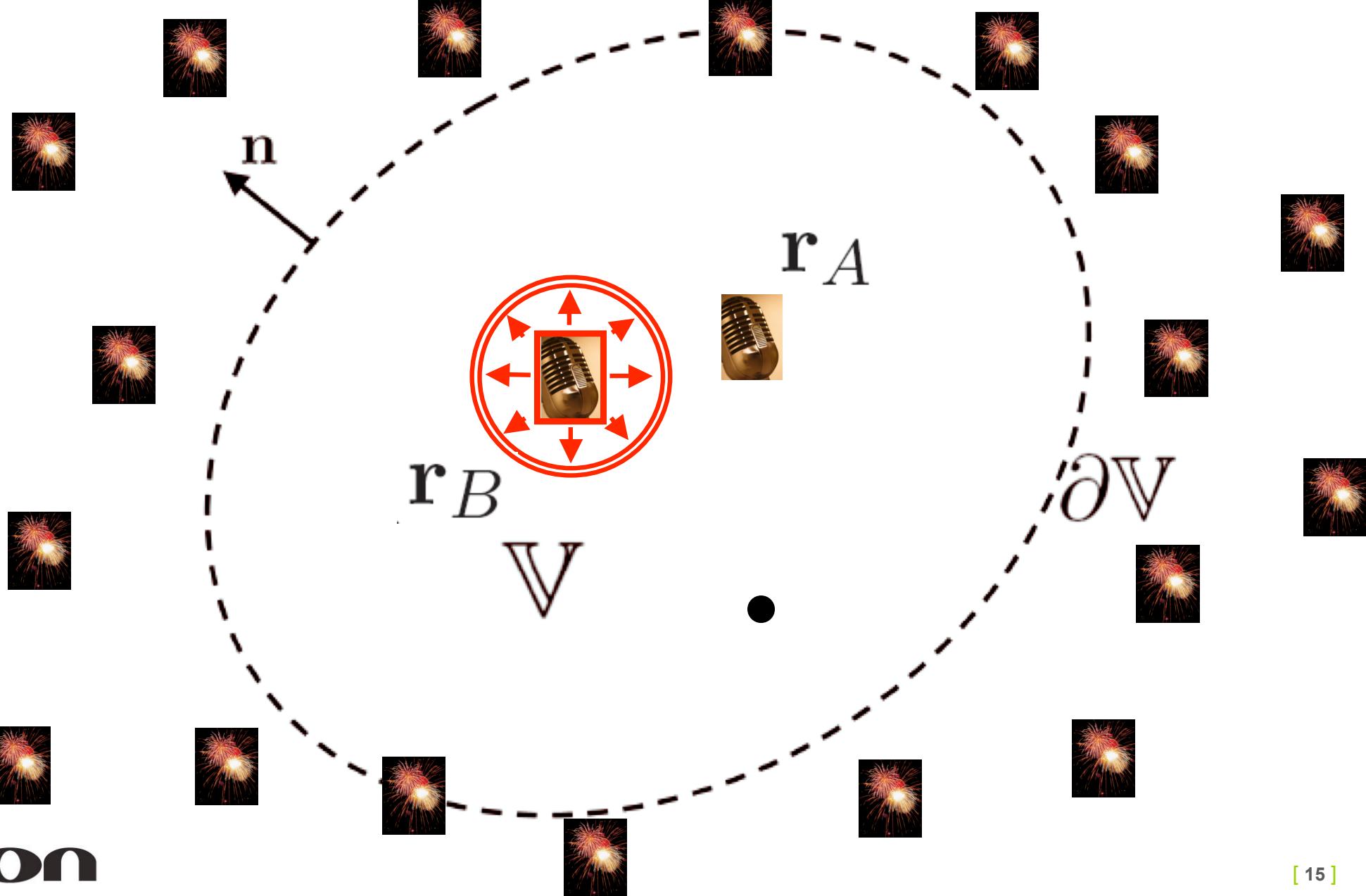
Interferometry - pseudo-source



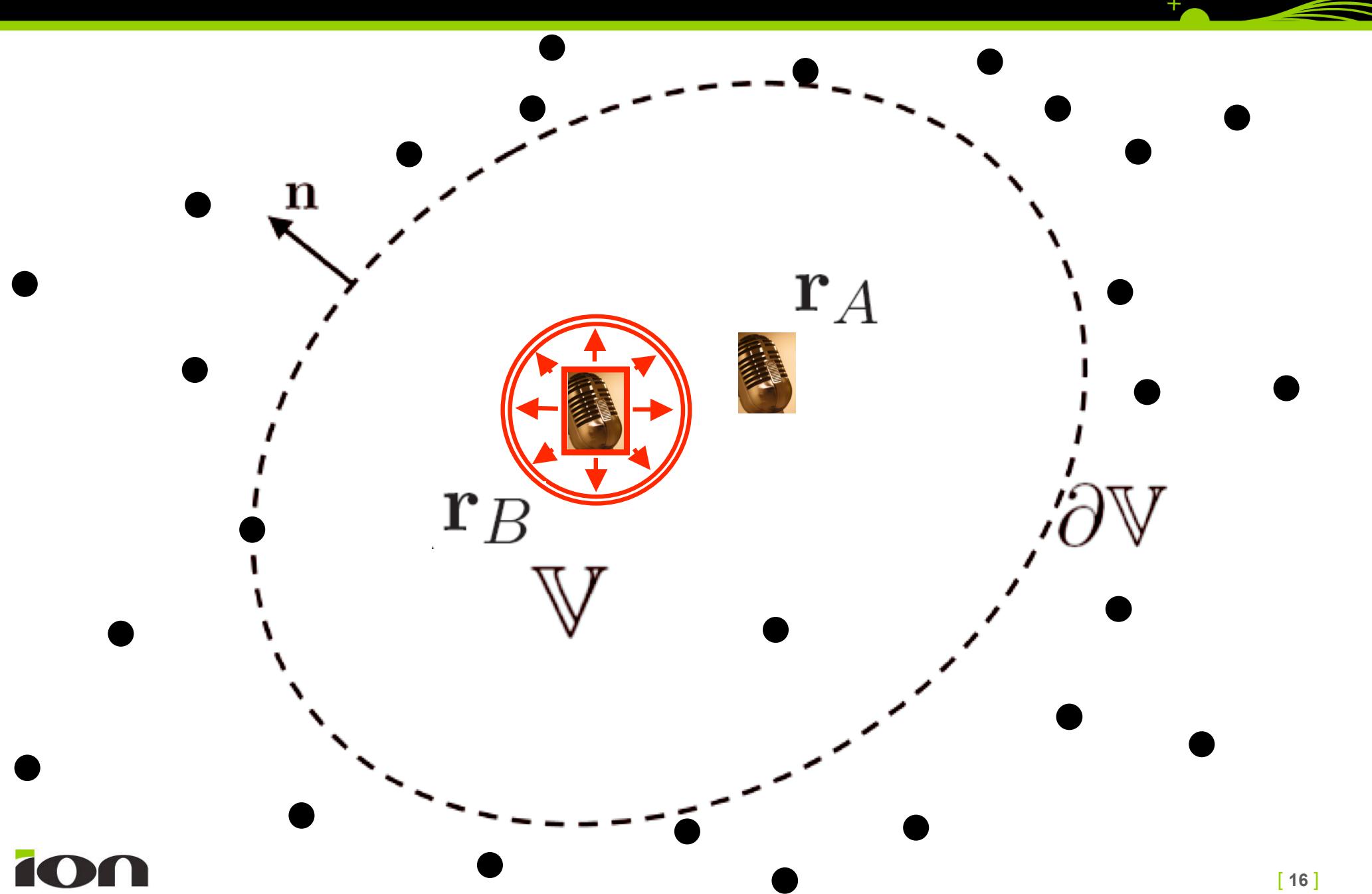
Interferometry - full wavefield



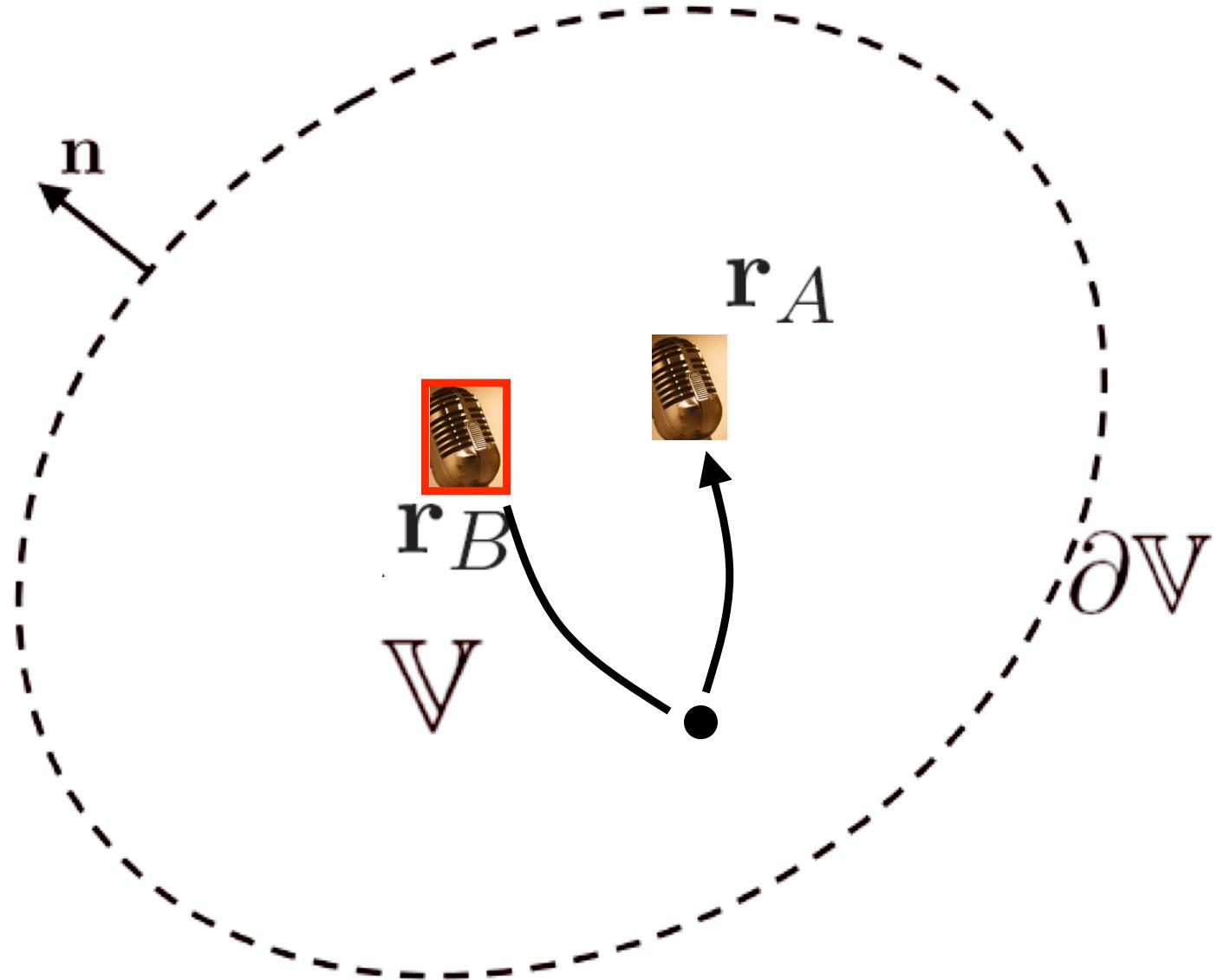
Condition: equipartitioning



Equipartitioning?



But what if... ?



Scattering: perturbed media



Perturbed media

$$\hat{\mathbf{A}} = \hat{\mathbf{A}}_0 + \hat{\mathbf{A}}_S,$$
$$\hat{\mathbf{B}} = \hat{\mathbf{B}}_0 + \hat{\mathbf{B}}_S$$

Perturbed fields

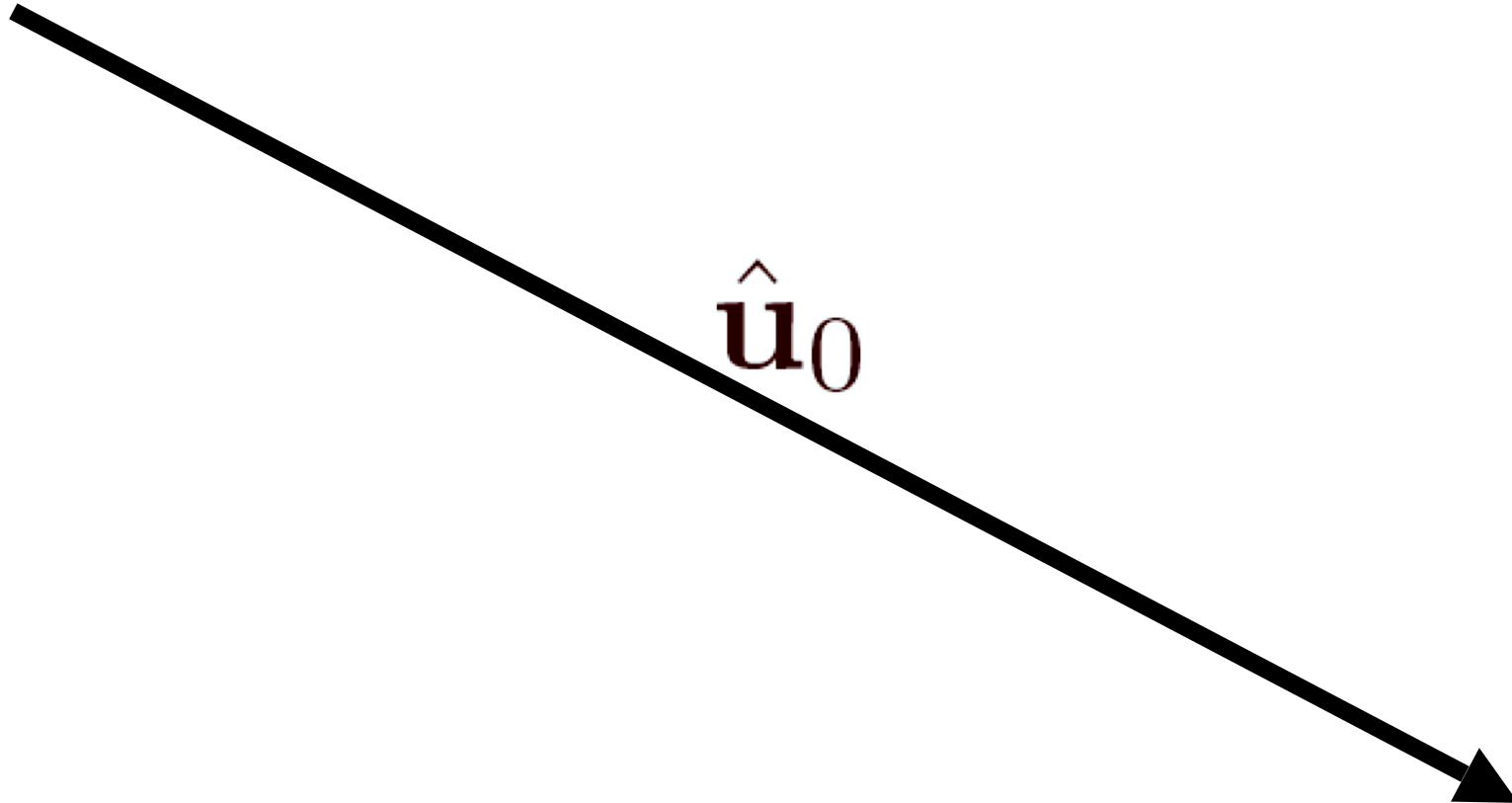
$$\hat{\mathbf{u}} = \hat{\mathbf{u}}_0 + \hat{\mathbf{u}}_S$$

Field perturbations = scattering

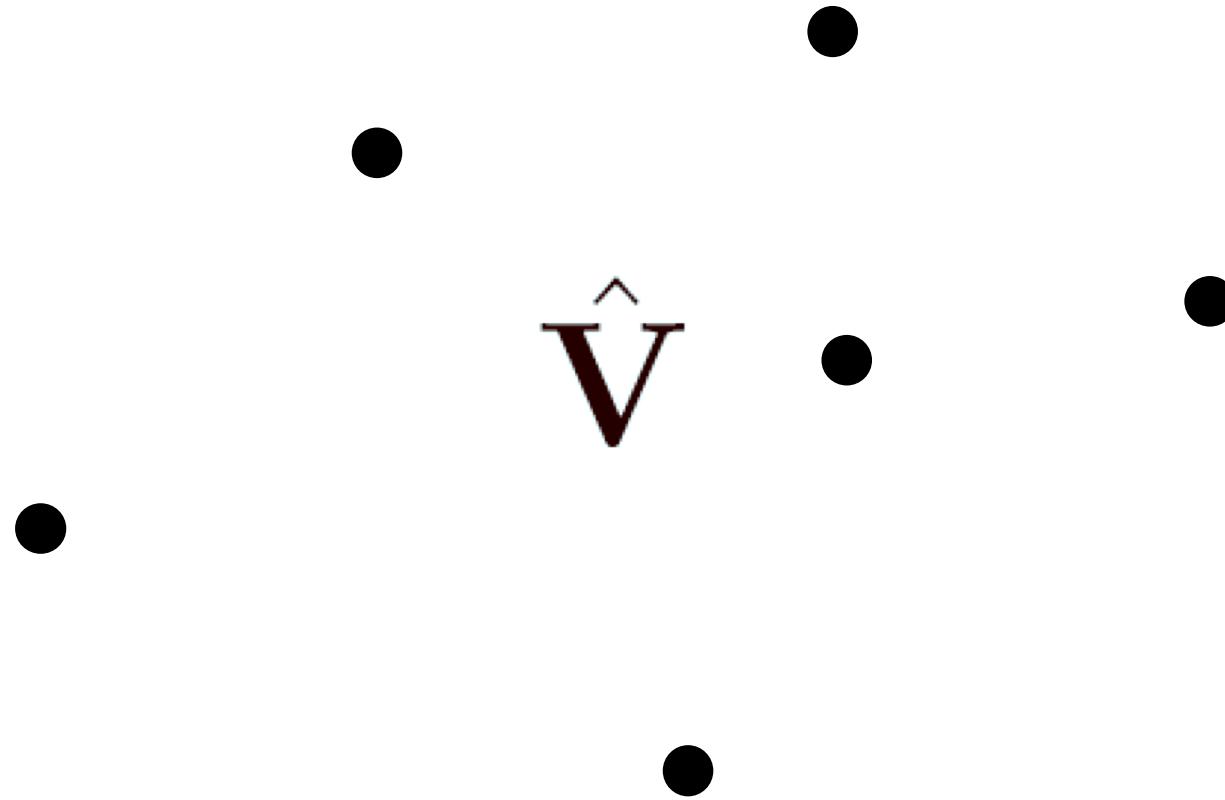
$$\hat{\mathbf{V}}\hat{\mathbf{u}}_0 = \hat{\mathbf{L}}\hat{\mathbf{u}}_S$$

Scattering Potential

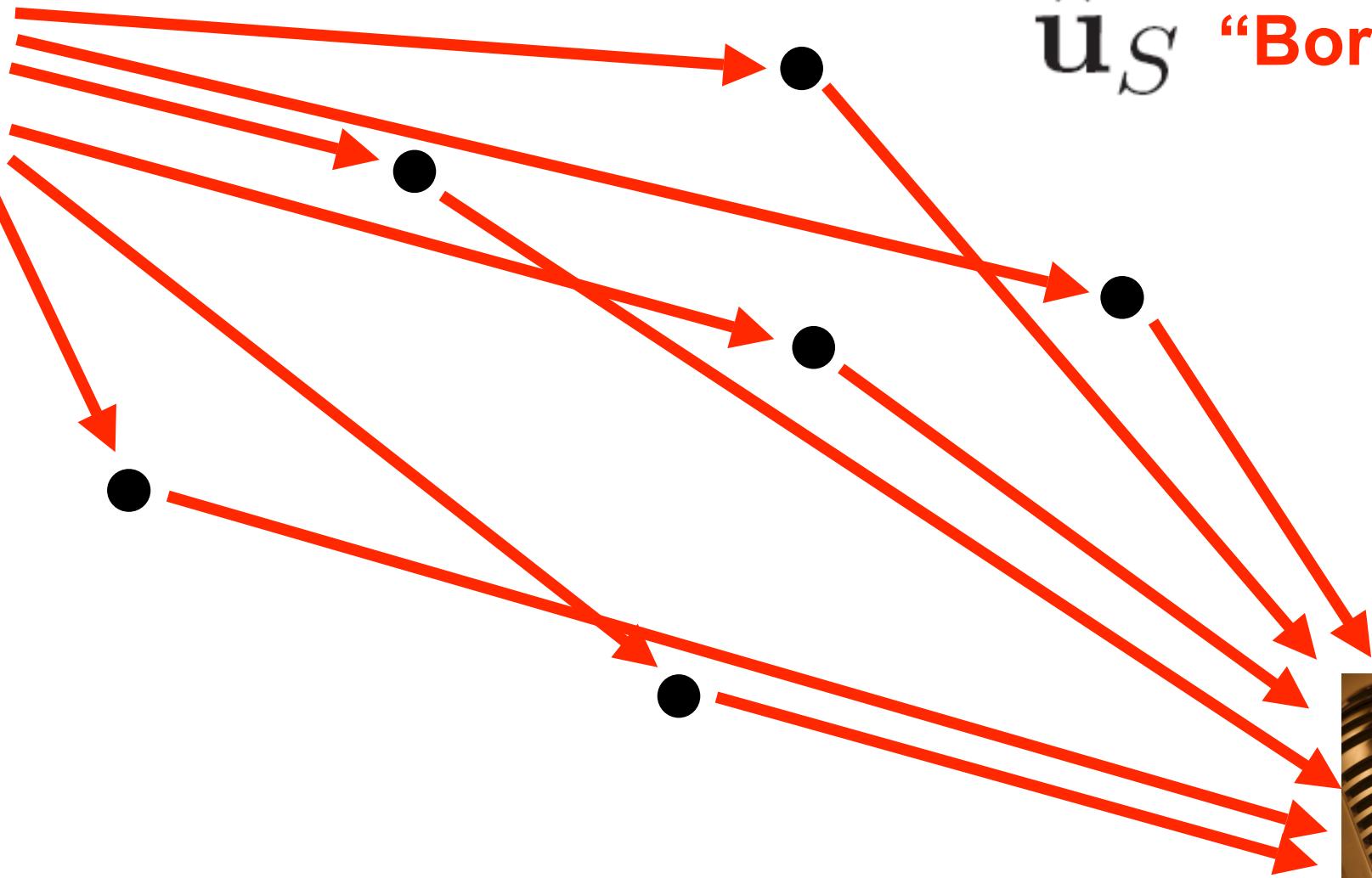
Scattering: example



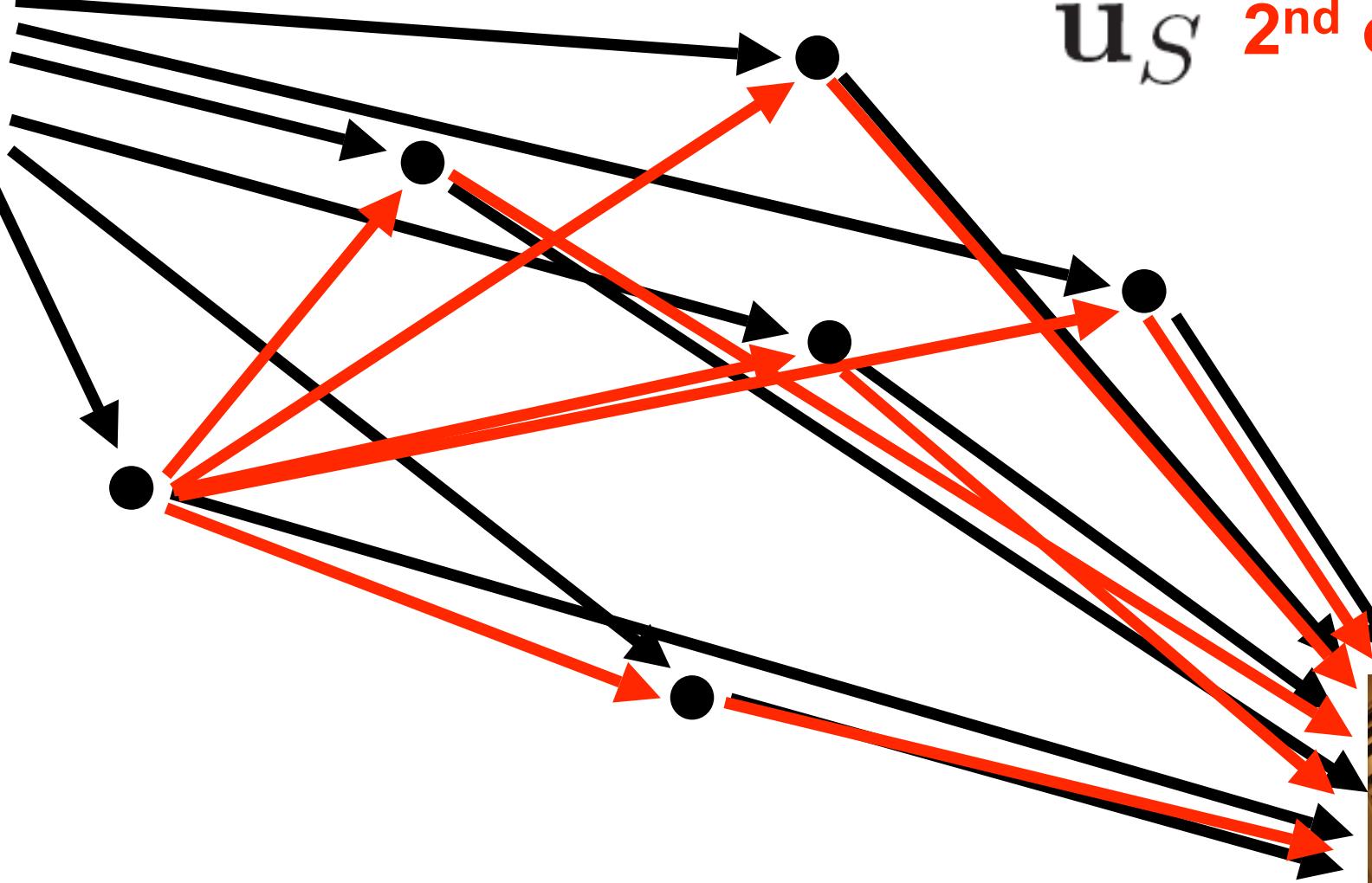
Scattering: potential



Scattered waves: perturbations



Scattered waves: multiple scattering



Perturbed field equations

$$\nabla p_0^A(\mathbf{r}, \omega) - i\omega\rho_0(\mathbf{r})\mathbf{v}_0^A(\mathbf{r}, \omega) = 0$$

$$\nabla \cdot \mathbf{v}_0^A(\mathbf{r}, \omega) - i\omega\kappa_0(\mathbf{r})p_0^A(\mathbf{r}, \omega) = q^A(\mathbf{r}, \omega)$$

$$\nabla p^A(\mathbf{r}, \omega) - i\omega\rho(\mathbf{r})\mathbf{v}^A(\mathbf{r}, \omega) = 0$$

$$\nabla \cdot \mathbf{v}^A(\mathbf{r}, \omega) - i\omega\kappa(\mathbf{r})p^A(\mathbf{r}, \omega) = q^A(\mathbf{r}, \omega)$$

Convolution theorem



$$\oint_{\mathbf{r} \in \partial \mathbb{V}} [p_S^A \mathbf{v}_0^B - p_0^B \mathbf{v}_S^A] \cdot d\mathbf{S} = \int_{\mathbf{r} \in \mathbb{V}} p_S^A q_0^B dV + \int_{\mathbf{r} \in \mathbb{V}} i\omega(\kappa_0 - \kappa) p^A p_0^B dV$$

Using Green's functions

$$q^{A,B} = \delta(\mathbf{r} - \mathbf{r}_{A,B})$$

$$p^{A,B}(\mathbf{r}, \omega) = G(\mathbf{r}, \mathbf{r}_{A,B}, \omega) = G_0(\mathbf{r}, \mathbf{r}_{A,B}, \omega) + G_S(\mathbf{r}, \mathbf{r}_{A,B}, \omega)$$

Convolution theorem

$$\int_{\mathbf{r} \in \mathbb{V}} G_S(\mathbf{r}, \mathbf{r}_A) \delta(\mathbf{r} - \mathbf{r}_B) dV = \int_{\mathbf{r} \in \mathbb{V}} i\omega(\kappa_0 - \kappa) G(\mathbf{r}, \mathbf{r}_A) G_0(\mathbf{r}, \mathbf{r}_B) dV$$
$$G_S(\mathbf{r}_B, \mathbf{r}_A) = \int_{\mathbf{r} \in \mathbb{V}} \frac{1}{i\omega\rho} G(\mathbf{r}, \mathbf{r}_A) V(\mathbf{r}) G_0(\mathbf{r}, \mathbf{r}_B) dV;$$

With homogeneous conditions on $\partial\mathbb{V}$

Lippmann-Schwinger: scattering!

Scattering: unperturbed (reference)

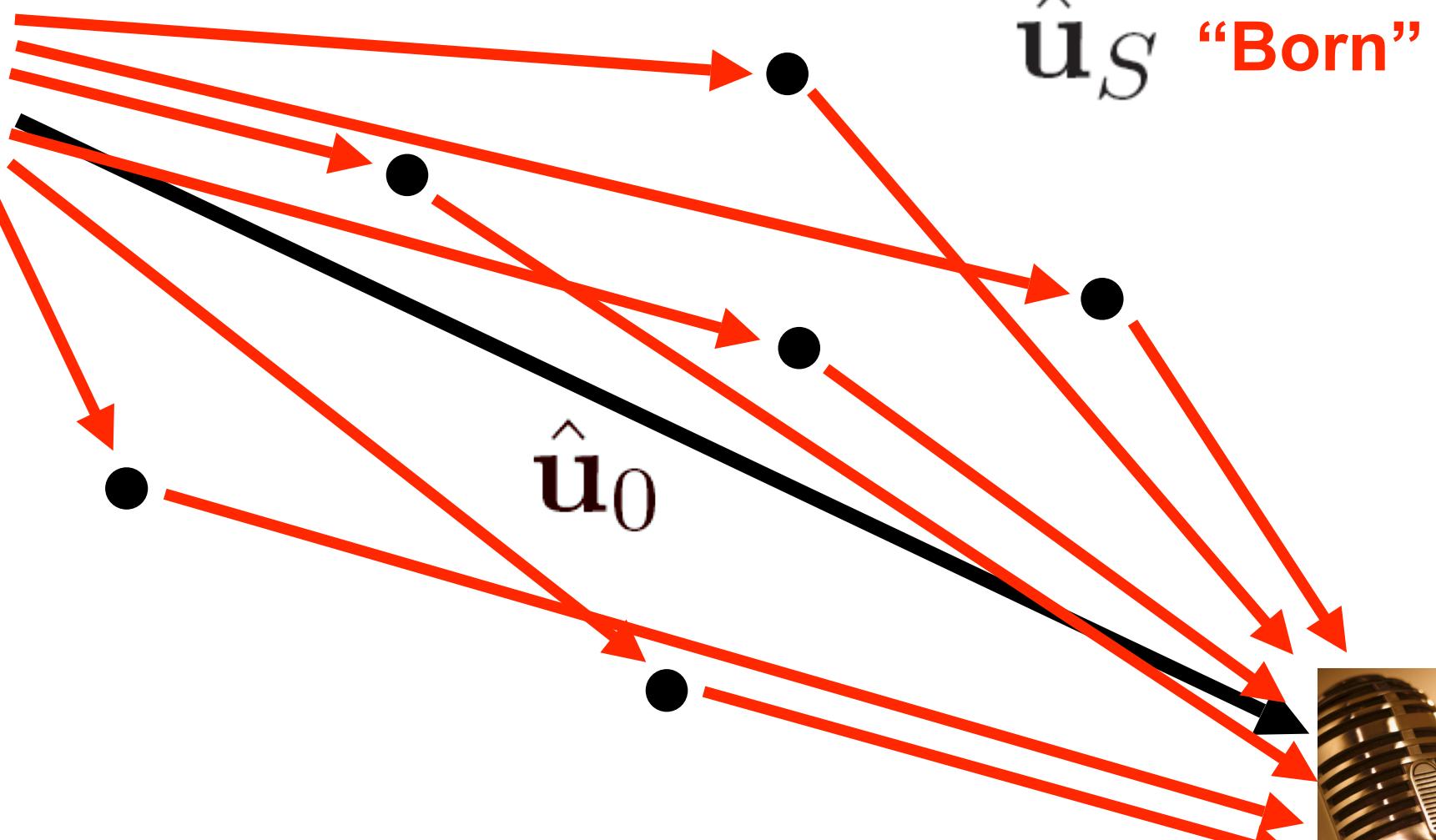
+



$\hat{\mathbf{u}}_0$



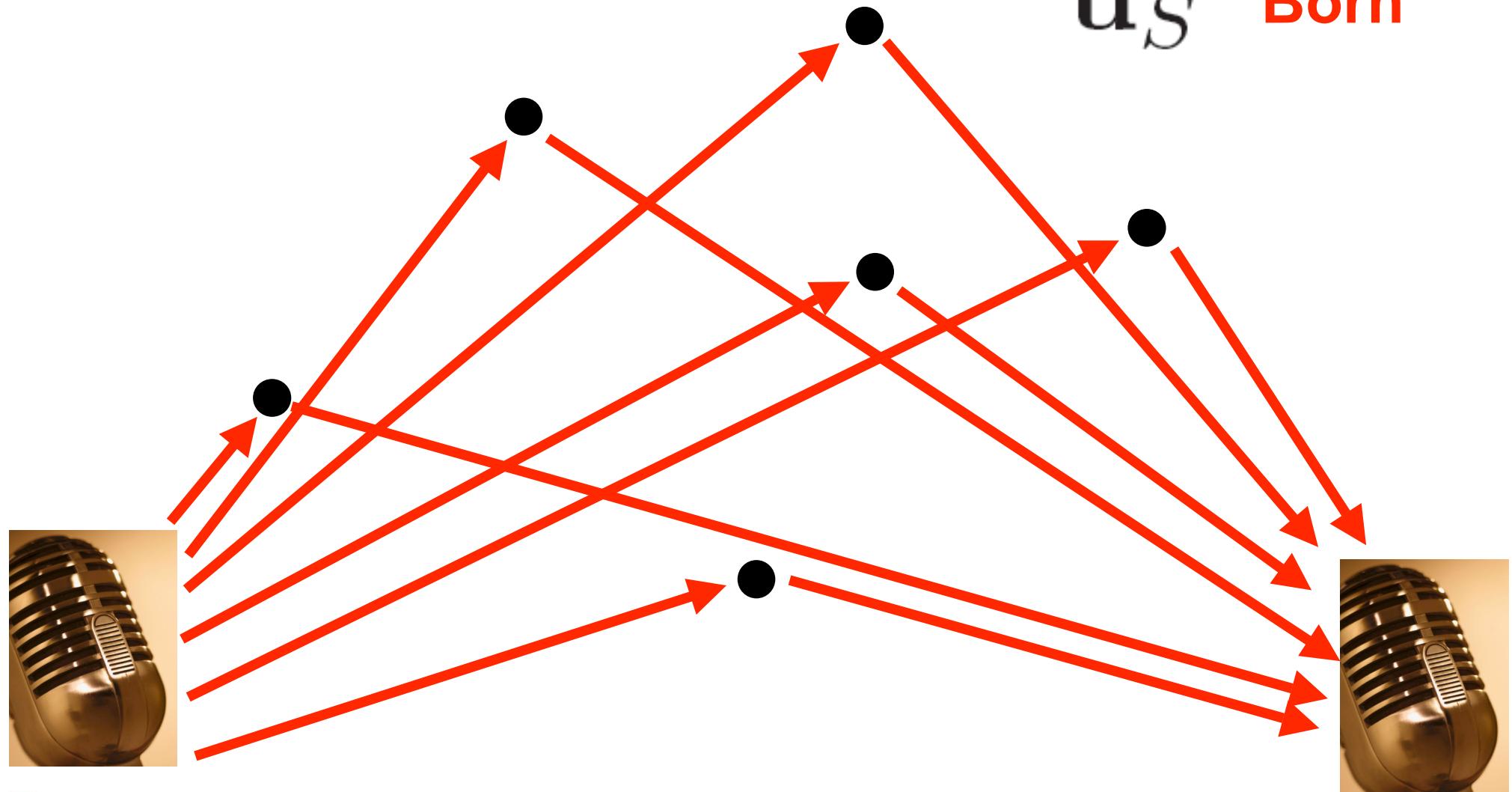
Scattered waves: perturbations



Scattered waves: perturbations

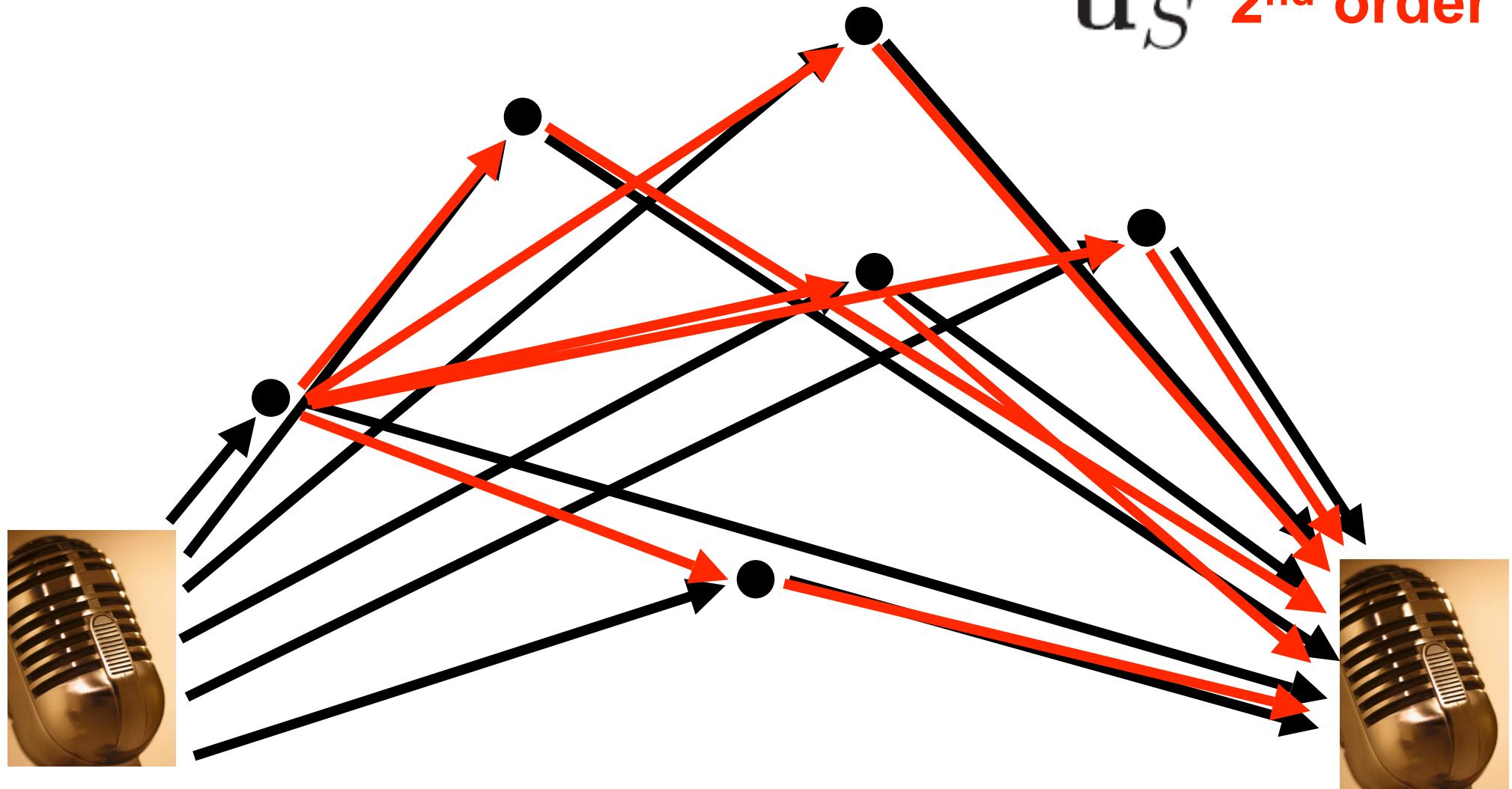


$\hat{\mathbf{u}}_S$ “Born”



Multiple scattering

$\hat{\mathbf{u}}_S$ 2nd order



Correlation theorem

+

$$\oint_{\mathbf{r} \in \partial \mathbb{V}} [p_S^A \mathbf{v}_0^{B*} + p_0^{B*} \mathbf{v}_S^A] \cdot d\mathbf{S} = \int_{\mathbf{r} \in \mathbb{V}} p_S^A q_0^{B*} dV - \int_{\mathbf{r} \in \mathbb{V}} i\omega(\kappa_0 - \kappa) p^A p_0^{B*} dV$$



$$\begin{aligned} G_S(\mathbf{r}_B, \mathbf{r}_A) &= \int_{\mathbf{r} \in \mathbb{V}} G_S(\mathbf{r}, \mathbf{r}_A) \delta(\mathbf{r} - \mathbf{r}_B) dV \\ &= \oint_{\mathbf{r} \in \partial \mathbb{V}} \frac{1}{i\omega\rho} [G_0^*(\mathbf{r}, \mathbf{r}_B) \nabla G_S(\mathbf{r}, \mathbf{r}_A) - G_S(\mathbf{r}, \mathbf{r}_A) \nabla G_0^*(\mathbf{r}, \mathbf{r}_B)] \cdot d\mathbf{S} \\ &+ \int_{\mathbf{r} \in \mathbb{V}} \frac{1}{i\omega\rho} G(\mathbf{r}, \mathbf{r}_A) \mathcal{V}(\mathbf{r}) G_0^*(\mathbf{r}, \mathbf{r}_B) dV . \end{aligned}$$

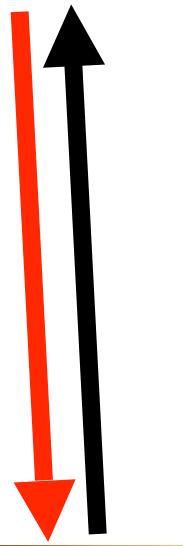
Volume integral

$$G_S(\mathbf{r}_B, \mathbf{r}_A) = \int_{\mathbf{r} \in \mathbb{V}} \frac{1}{i\omega\rho} G(\mathbf{r}, \mathbf{r}_A) \mathcal{V}(\mathbf{r}) G_0^*(\mathbf{r}, \mathbf{r}_B) dV$$

Similar to Lippmann-Schwinger...

ONLY under Dirichlet or Neumann B.C.

Scattering in lossless media



$$\hat{\mathbf{u}}_0$$

Reference waves can propagate either way!

Only volume integral

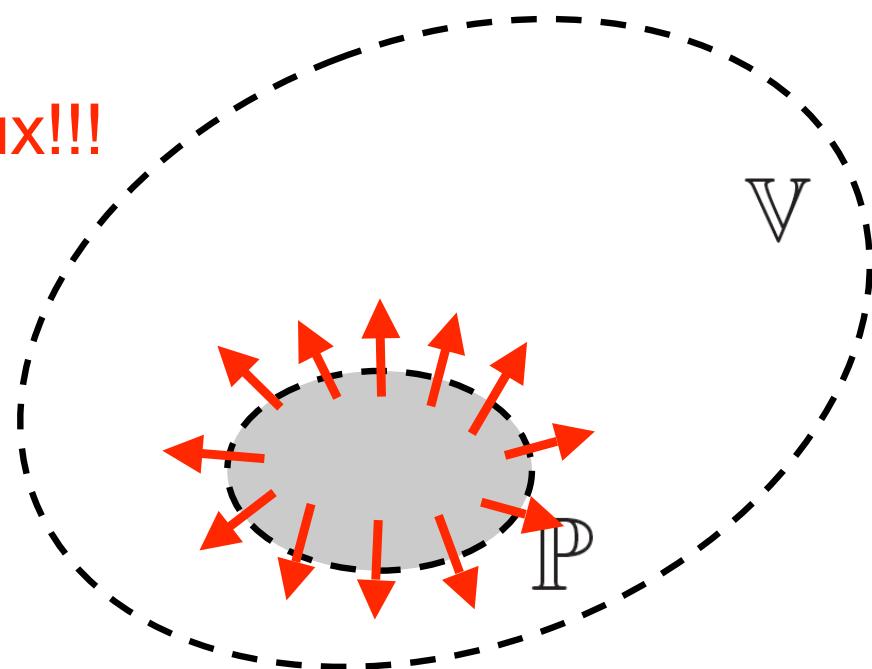


$$G_S(\mathbf{r}_B, \mathbf{r}_A) = \frac{i\omega}{|R(\omega)|^2} p(\mathbf{r}_A)p_0^*(\mathbf{r}_B)$$

$$q(\mathbf{r}_1, \omega)q^*(\mathbf{r}_2, \omega) = \Delta\kappa(\mathbf{r}_1, \omega)\delta(\mathbf{r}_1 - \mathbf{r}_2)|R(\omega)|^2$$

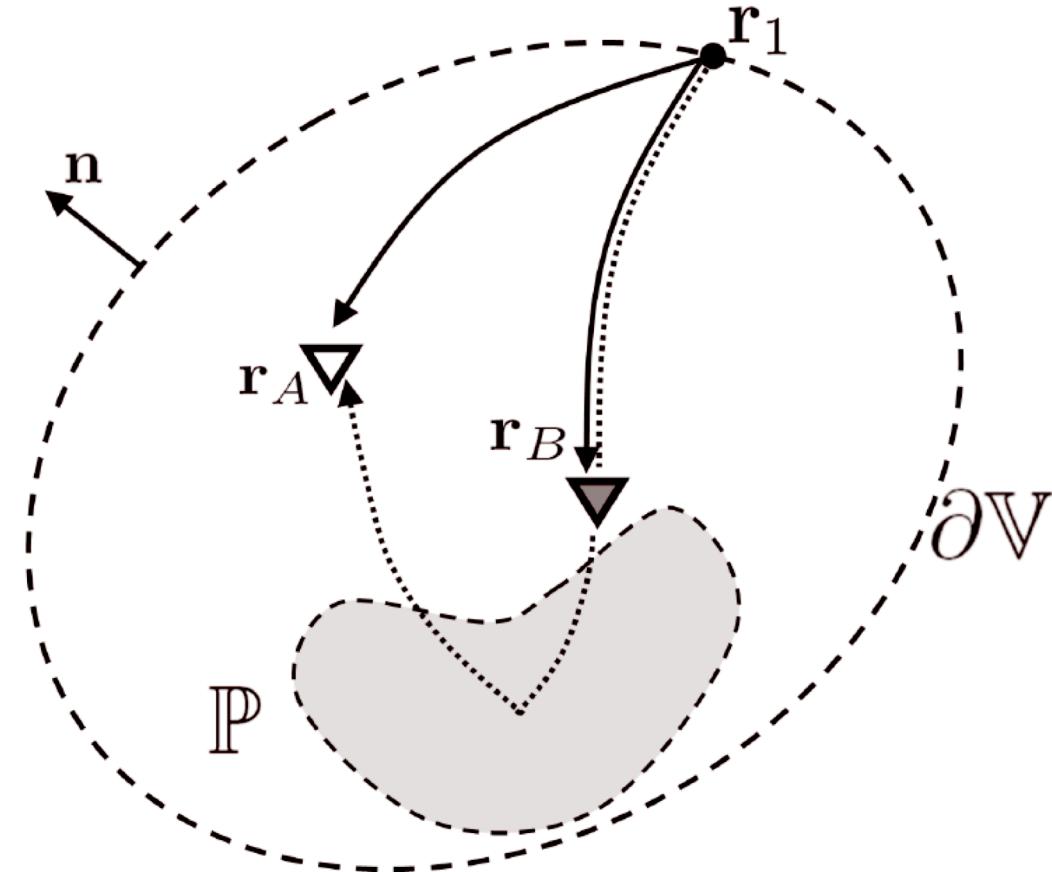
Random sources in \mathbb{V}

Nonzero net flux!!!

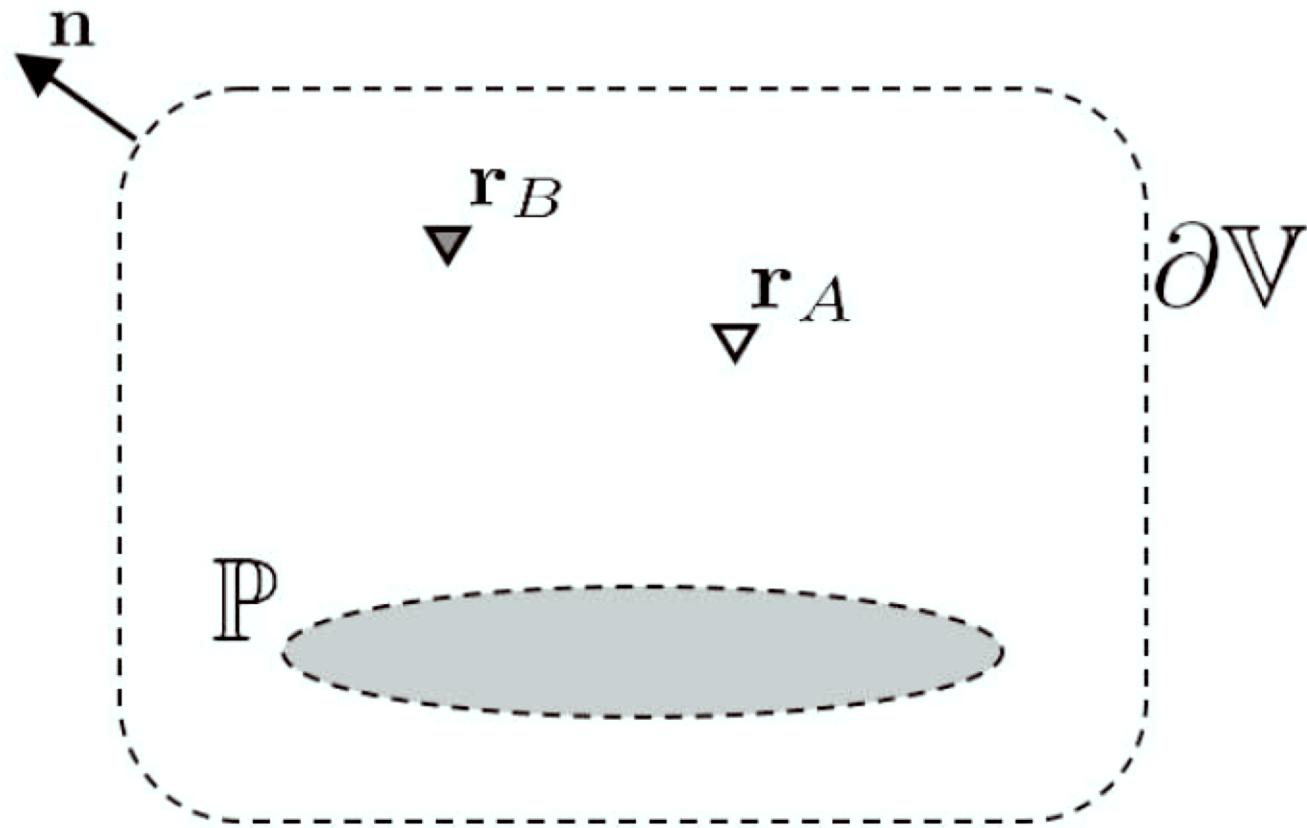


Surface & volume terms

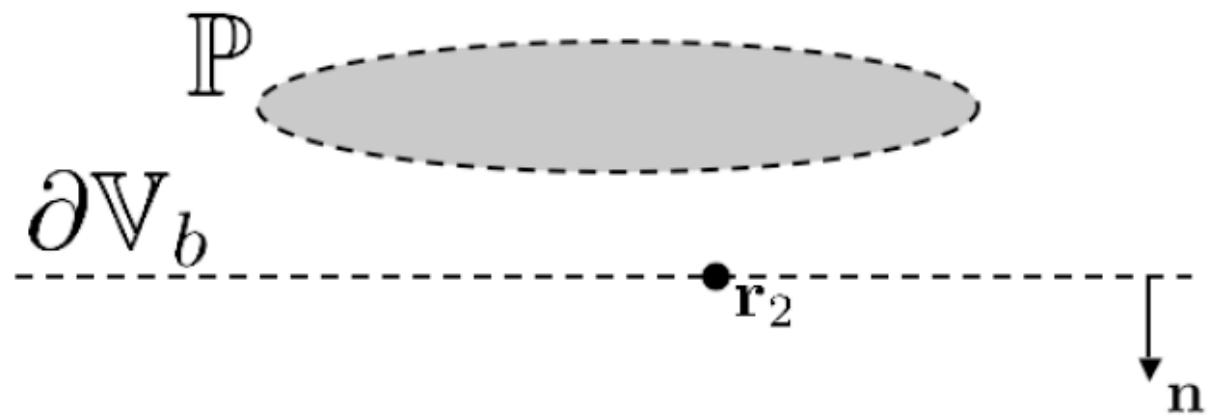
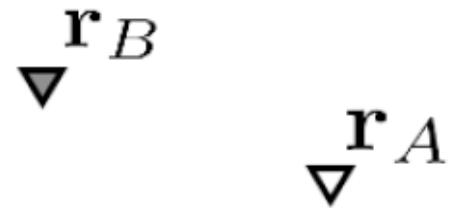
$$\begin{aligned} G_S(\mathbf{r}_B, \mathbf{r}_A) &= \int_{\mathbf{r} \in \mathbb{V}} G_S(\mathbf{r}, \mathbf{r}_A) \delta(\mathbf{r} - \mathbf{r}_B) dV \\ &= \oint_{\mathbf{r} \in \partial \mathbb{V}} \frac{1}{i\omega\rho} [G_0^*(\mathbf{r}, \mathbf{r}_B) \nabla G_S(\mathbf{r}, \mathbf{r}_A) - G_S(\mathbf{r}, \mathbf{r}_A) \nabla G_0^*(\mathbf{r}, \mathbf{r}_B)] \cdot d\mathbf{S} \\ &+ \int_{\mathbf{r} \in \mathbb{V}} \frac{1}{i\omega\rho} G(\mathbf{r}, \mathbf{r}_A) \mathcal{V}(\mathbf{r}) G_0^*(\mathbf{r}, \mathbf{r}_B) dV. \end{aligned}$$



Surface & volume terms

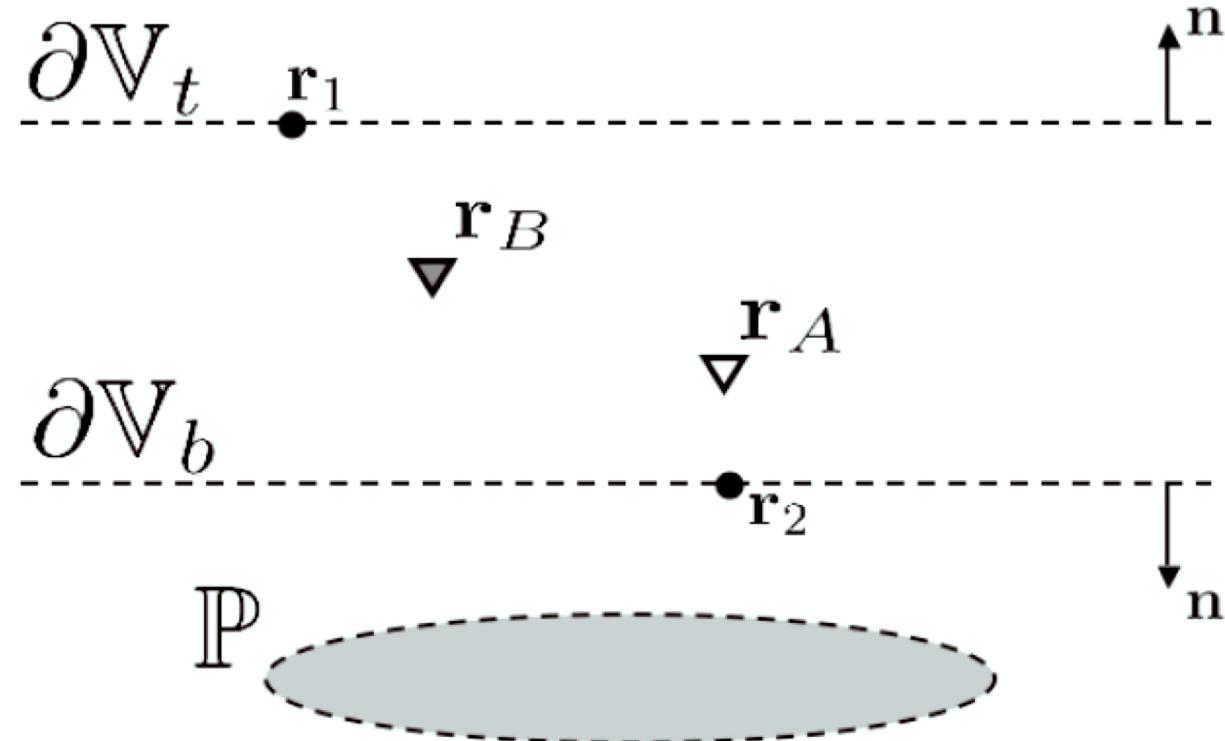


Surface & volume terms

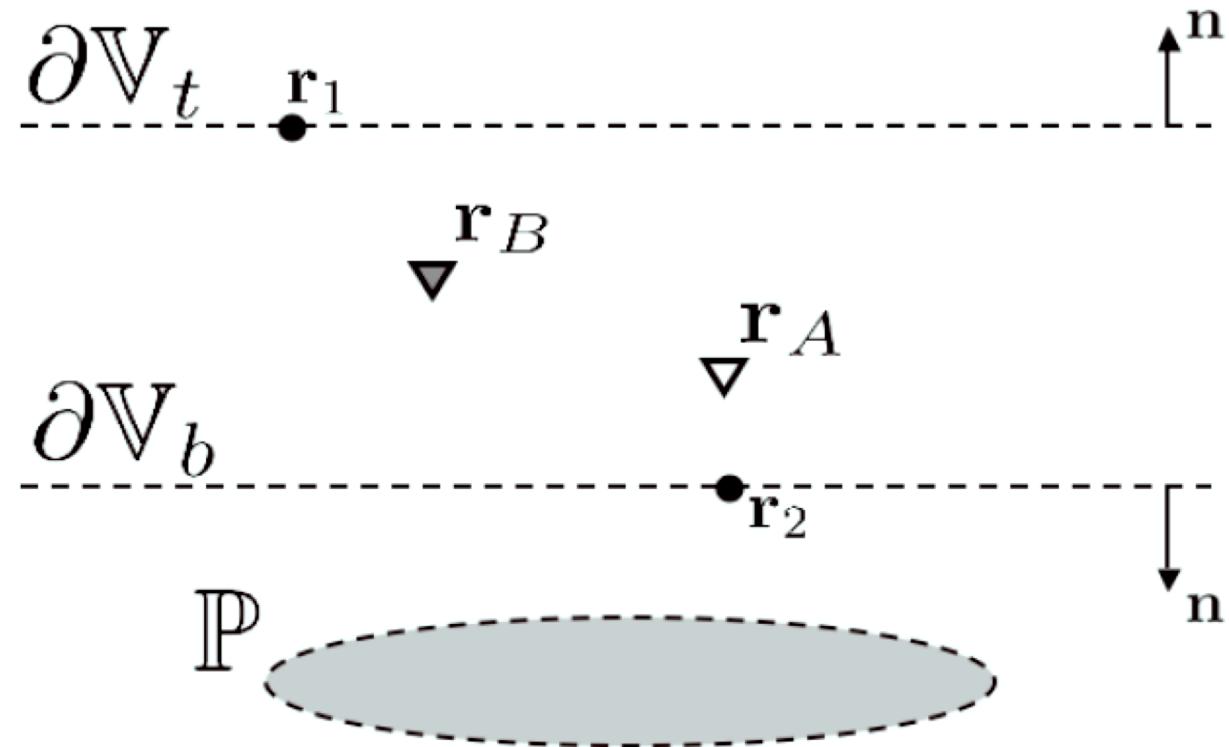


$$\begin{aligned}
 G_S(\mathbf{r}_B, \mathbf{r}_A) = & \int_{\mathbf{r} \in (\partial V_b \cup \partial V_t)} \frac{1}{i\omega\rho} [G_0^*(\mathbf{r}, \mathbf{r}_B) \nabla G_S(\mathbf{r}, \mathbf{r}_A) - G_S(\mathbf{r}, \mathbf{r}_A) \nabla G_0^*(\mathbf{r}, \mathbf{r}_B)] \cdot d\mathbf{S} \\
 & + \int_{\mathbf{r} \in \mathbb{P}} \frac{1}{i\omega\rho} G(\mathbf{r}, \mathbf{r}_A) \mathcal{V}(\mathbf{r}) G_0^*(\mathbf{r}, \mathbf{r}_B) dV ;
 \end{aligned} \tag{22}$$

Surface & volume terms

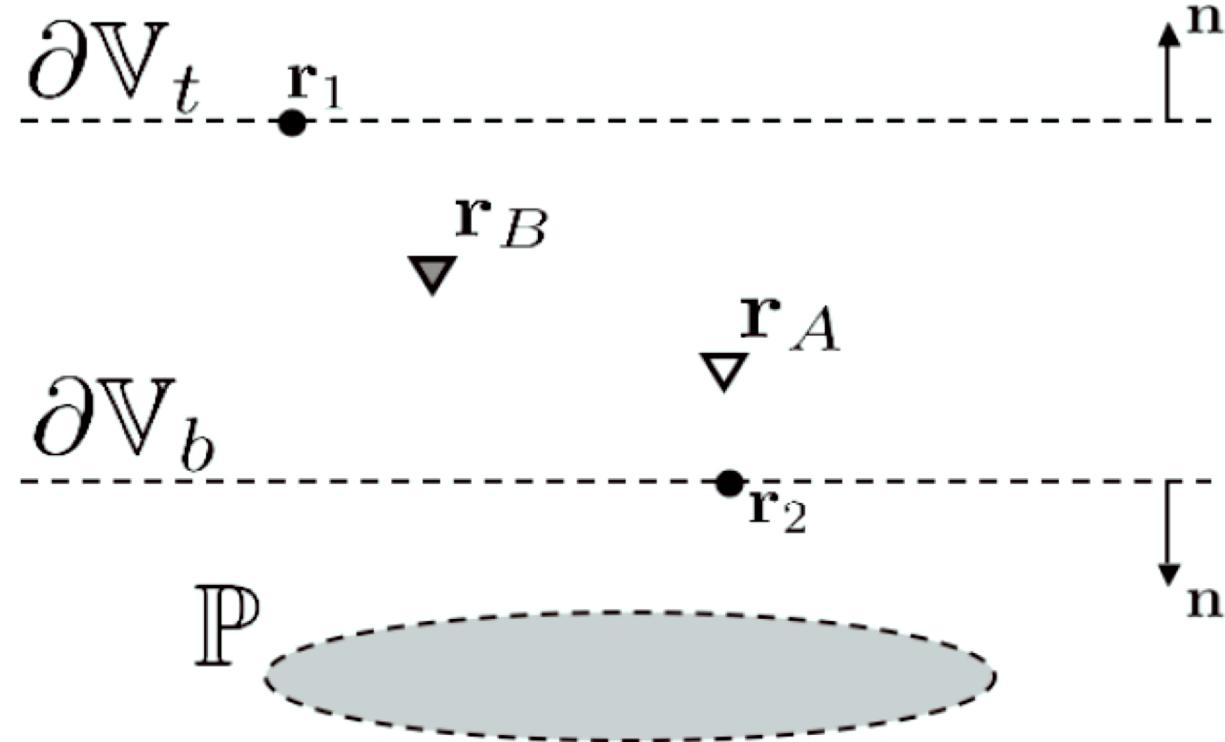


Surface & volume terms



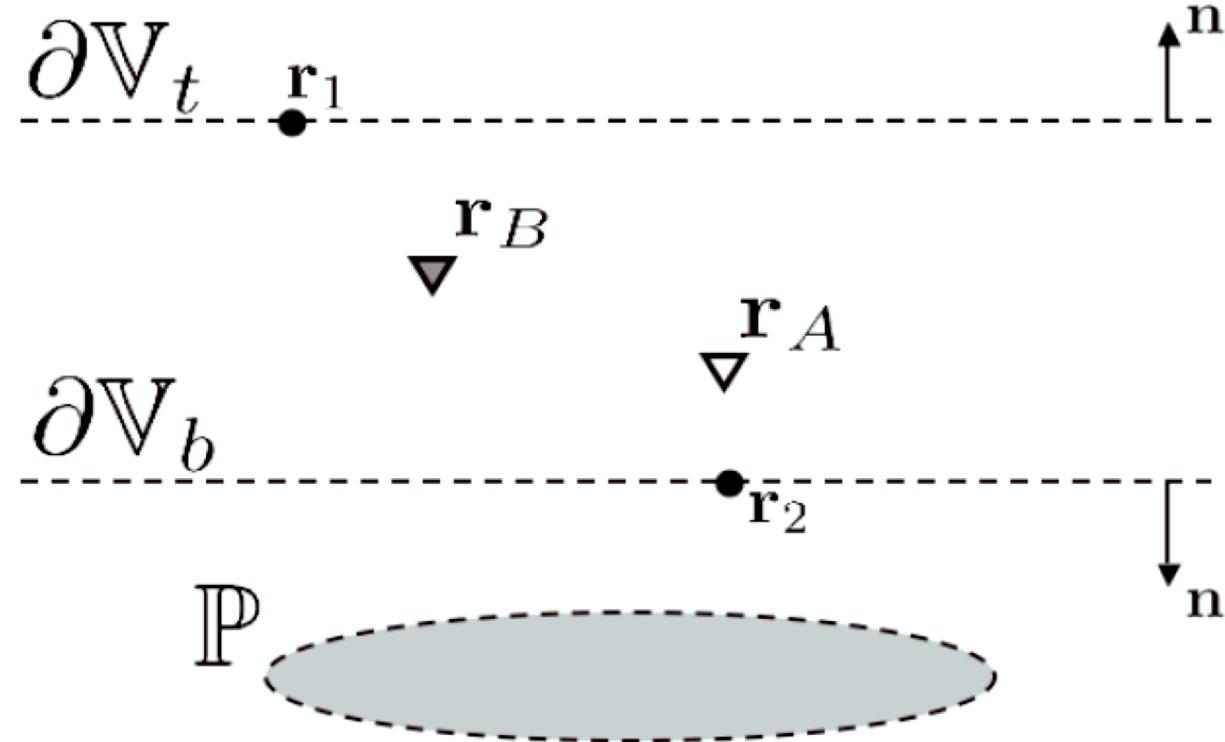
$$\int_{\mathbf{r} \in \mathbb{V}} \frac{1}{i\omega\rho} G(\mathbf{r}, \mathbf{r}_A) \mathcal{V}(\mathbf{r}) G_0^*(\mathbf{r}, \mathbf{r}_B) dV = 0$$

Surface & volume terms



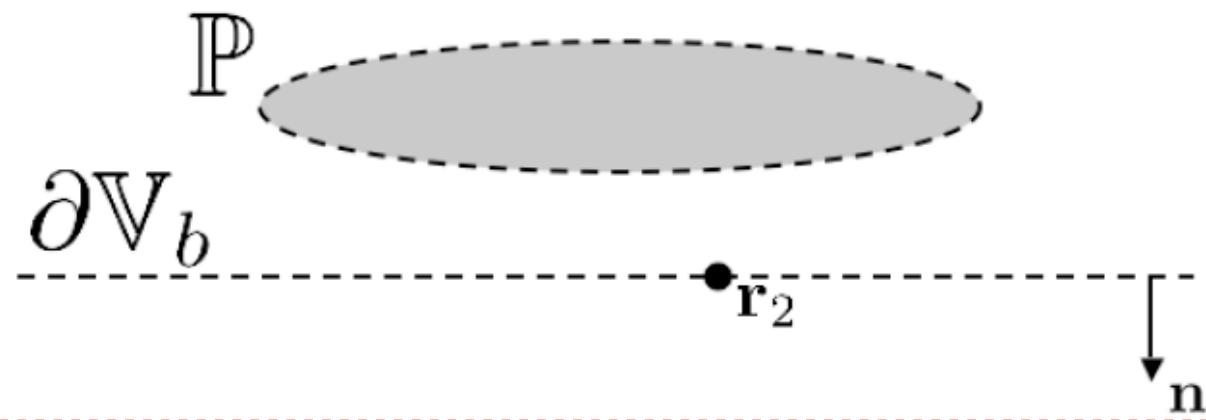
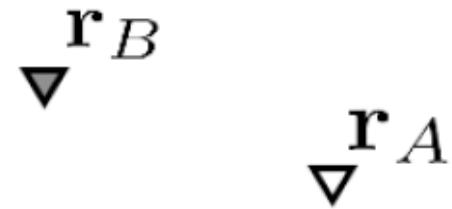
$$\int_{\mathbf{r} \in \partial\mathbb{V}_b} \frac{1}{i\omega\rho} [G_0^*(\mathbf{r}, \mathbf{r}_B) \nabla G_S(\mathbf{r}, \mathbf{r}_A) - G_S(\mathbf{r}, \mathbf{r}_A) \nabla G_0^*(\mathbf{r}, \mathbf{r}_B)] \cdot d\mathbf{S} = 0$$

Surface & volume terms



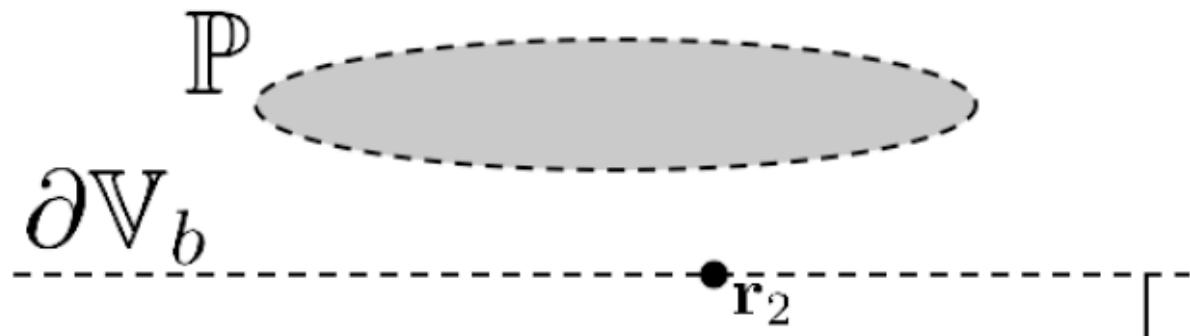
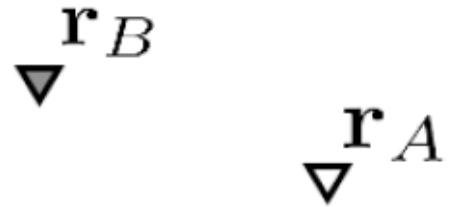
$$G_S(\mathbf{r}_B, \mathbf{r}_A) = \int_{\mathbf{r} \in \partial \mathbb{V}_t} \frac{1}{i\omega\rho} [G_S(\mathbf{r}, \mathbf{r}_A) \nabla G_0^*(\mathbf{r}, \mathbf{r}_B) + G_0^*(\mathbf{r}, \mathbf{r}_B) \nabla G_S(\mathbf{r}, \mathbf{r}_A)] \cdot d\mathbf{S}$$

Surface & volume terms



$$G_S(\mathbf{r}_B, \mathbf{r}_A) = \int_{\mathbf{r} \in \partial \mathbb{V}_t} \frac{1}{i\omega\rho} [G_S(\mathbf{r}, \mathbf{r}_A) \nabla G_0^*(\mathbf{r}, \mathbf{r}_B) + G_0^*(\mathbf{r}, \mathbf{r}_B) \nabla G_S(\mathbf{r}, \mathbf{r}_A)] \cdot d\mathbf{S}$$

Surface & volume terms



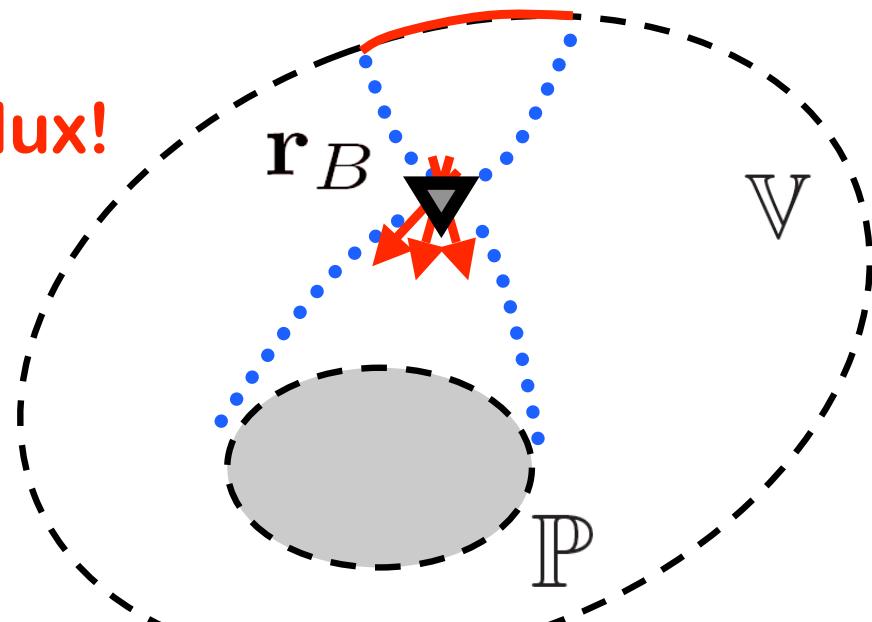
$$\int_{\mathbf{r} \in \partial \mathbb{V}_b} \frac{1}{i\omega\rho} [G_0^*(\mathbf{r}, \mathbf{r}_B) \nabla G_S(\mathbf{r}, \mathbf{r}_A) - G_S(\mathbf{r}, \mathbf{r}_A) \nabla G_0^*(\mathbf{r}, \mathbf{r}_B)] \cdot d\mathbf{S} = \\ - \int_{\mathbf{r} \in \mathbb{V}} \frac{1}{i\omega\rho} G(\mathbf{r}, \mathbf{r}_A) \mathcal{V}(\mathbf{r}) G_0^*(\mathbf{r}, \mathbf{r}_B) dV$$

Physical implications



- No energy equipartitioning; no dynamic equilibrium
- Attenuative scattering & transport: elastodynamics, electromagnetics, quantum mechanics, coupled systems

Nonzero net flux!



EXERCISE: terms



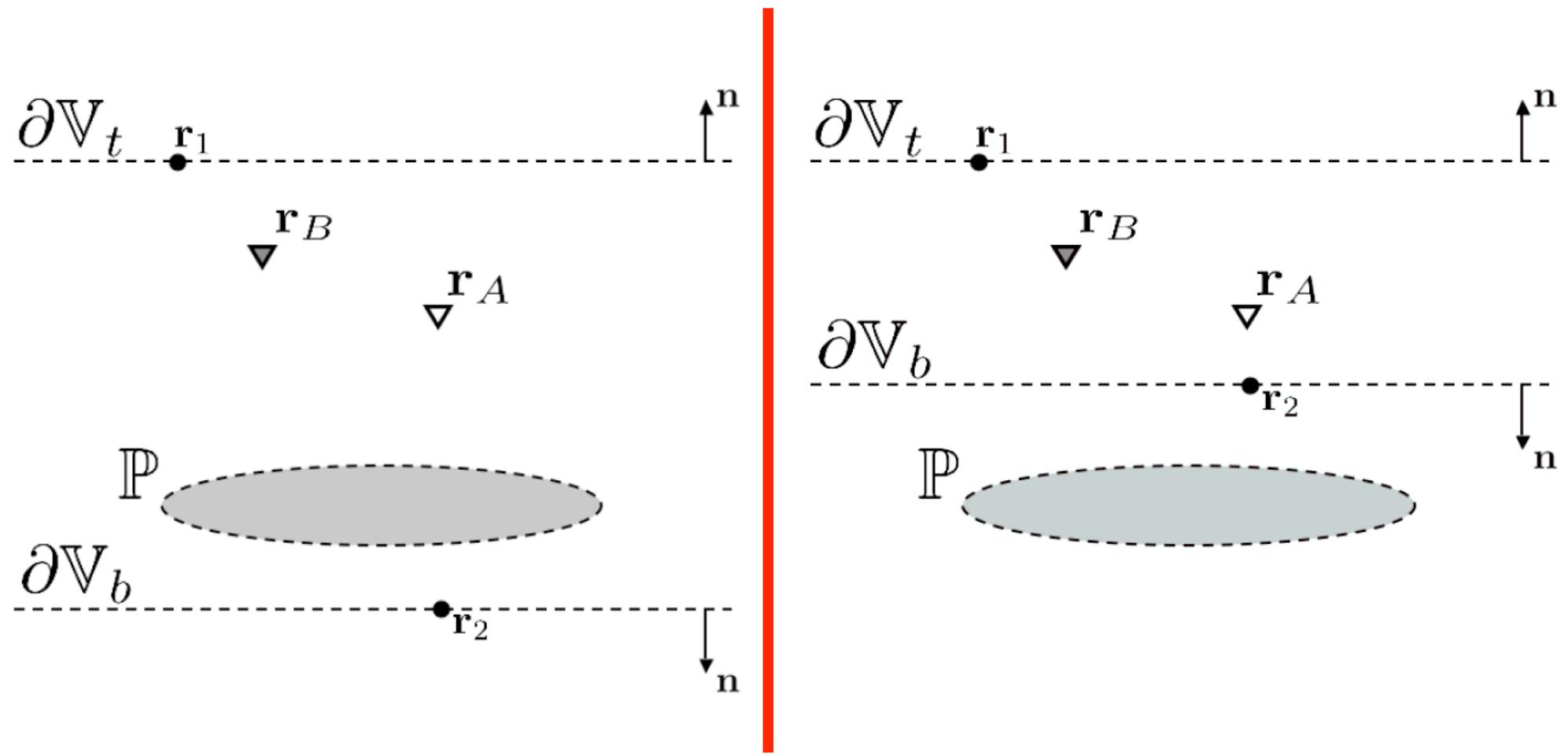
Full-wavefield interferometry

$$\oint_{\mathbf{r} \in \partial \mathbb{V}} [p^A \mathbf{v}^{B*} + p^{B*} \mathbf{v}^A] \cdot d\mathbf{S} = \int_{\mathbf{r} \in \mathbb{V}} [p^A q^{B*} + p^{B*} q^A] dV$$

Scattered-field interferometry

$$\oint_{\mathbf{r} \in \partial \mathbb{V}} [p_S^A \mathbf{v}_0^{B*} + p_0^{B*} \mathbf{v}_S^A] \cdot d\mathbf{S} = \int_{\mathbf{r} \in \mathbb{V}} p_S^A q_0^{B*} dV - \int_{\mathbf{r} \in \mathbb{V}} i\omega(\kappa_0 - \kappa) p^A p_0^{B*} dV$$

EXERCISE: setup





+

Correlation interferometry

$$\oint_{\partial \mathbb{V}} C_{AB} d\mathbf{s} = \langle |W(\mathbf{s})|^2 \rangle [G(\mathbf{r}_A, \mathbf{r}_B) + G^*(\mathbf{r}_A, \mathbf{r}_B)]$$

- causal & anticausal
- source spectra

Deconvolution interferometry



$$D_{AB} = \frac{u(\mathbf{r}_A, \mathbf{s})}{u(\mathbf{r}_B, \mathbf{s})} = \frac{u(\mathbf{r}_A, \mathbf{s}) u^*(\mathbf{r}_B, \mathbf{s})}{|u(\mathbf{r}_B, \mathbf{s})|^2} = \frac{G(\mathbf{r}_A, \mathbf{s}) G^*(\mathbf{r}_B, \mathbf{s})}{|G(\mathbf{r}_B, \mathbf{s})|^2}$$

$$D_{AB} = \frac{G(\mathbf{r}_A, \mathbf{s})}{G_0(\mathbf{r}_B, \mathbf{s})} \sum_{n=0}^{\infty} (-1)^n \left(\frac{G_S(\mathbf{r}_B, \mathbf{s})}{G_0(\mathbf{r}_B, \mathbf{s})} \right)^n$$

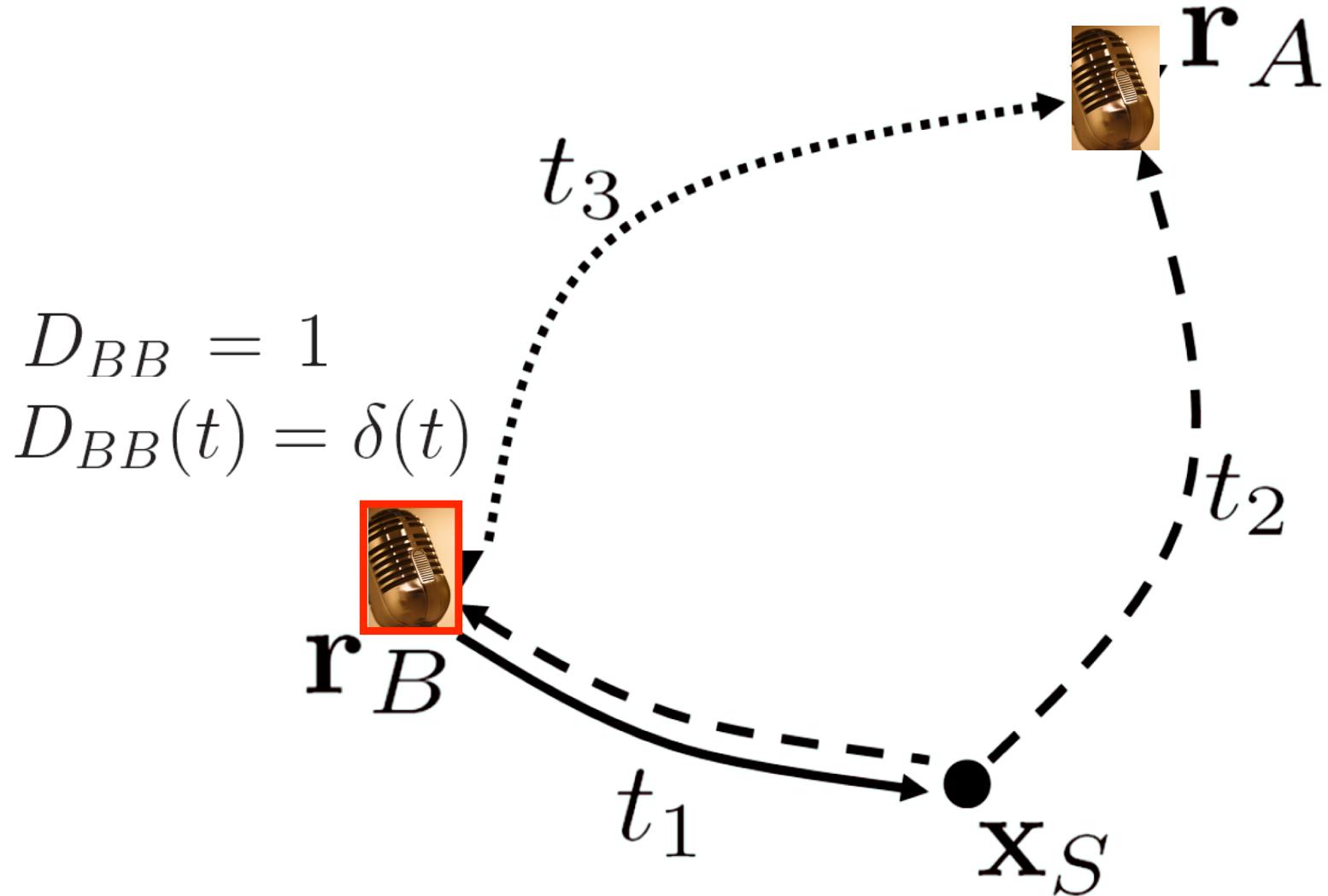
- causal*
- no source
- many terms...

Deconvolution interferometry

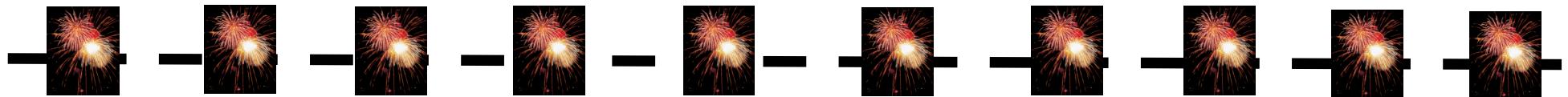


$$\oint_{\partial \mathbb{V}} D_{AB} d\mathbf{s} = \underbrace{\oint_{\partial \mathbb{V}} \frac{G_0(\mathbf{r}_A, \mathbf{s}) G_0^*(\mathbf{r}_B, \mathbf{s})}{|G_0(\mathbf{r}_B, \mathbf{s})|^2} d\mathbf{s}}_{D_{AB}^1} + \underbrace{\oint_{\partial \mathbb{V}} \frac{G_S(\mathbf{r}_A, \mathbf{s}) G_0^*(\mathbf{r}_B, \mathbf{s})}{|G_0(\mathbf{r}_B, \mathbf{s})|^2} d\mathbf{s}}_{D_{AB}^2} - \underbrace{\oint_{\partial \mathbb{V}} \frac{G_0(\mathbf{r}_A, \mathbf{s}) G_0^*(\mathbf{r}_B, \mathbf{s}) G_S(\mathbf{r}_B, \mathbf{s})}{|G_0(\mathbf{r}_B, \mathbf{s})|^2 G_0(\mathbf{r}_B, \mathbf{s})} d\mathbf{s}}_{D_{AB}^3}.$$

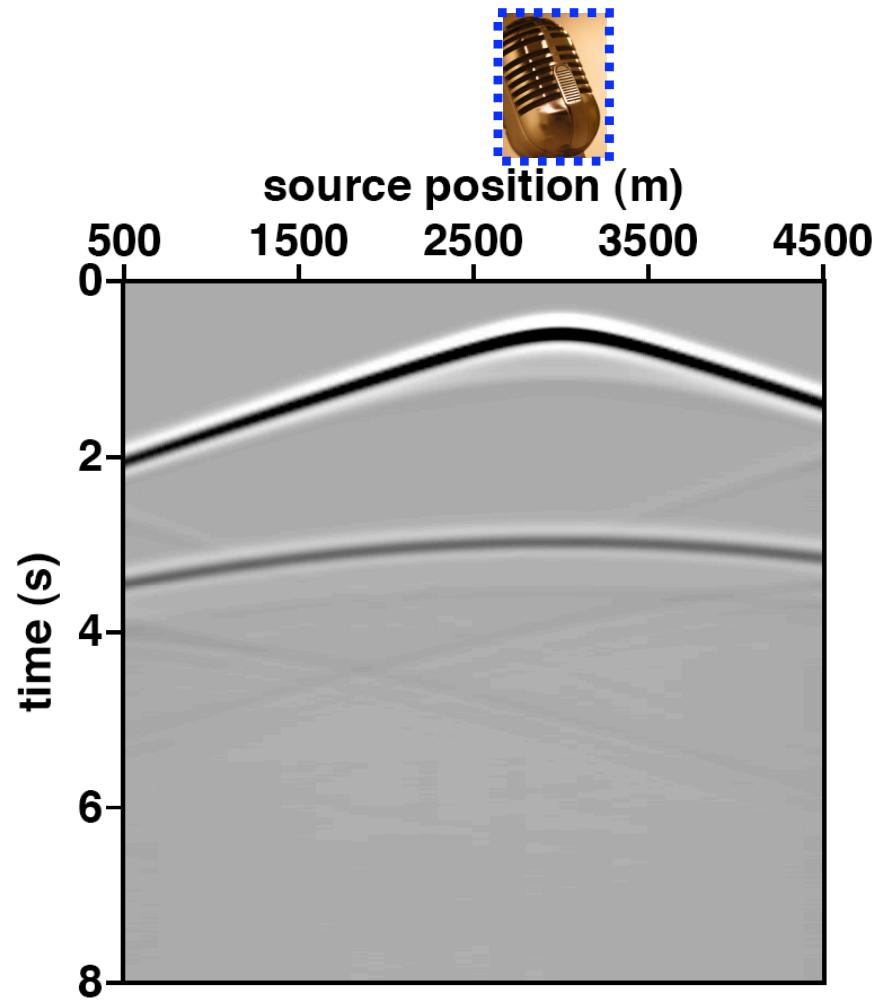
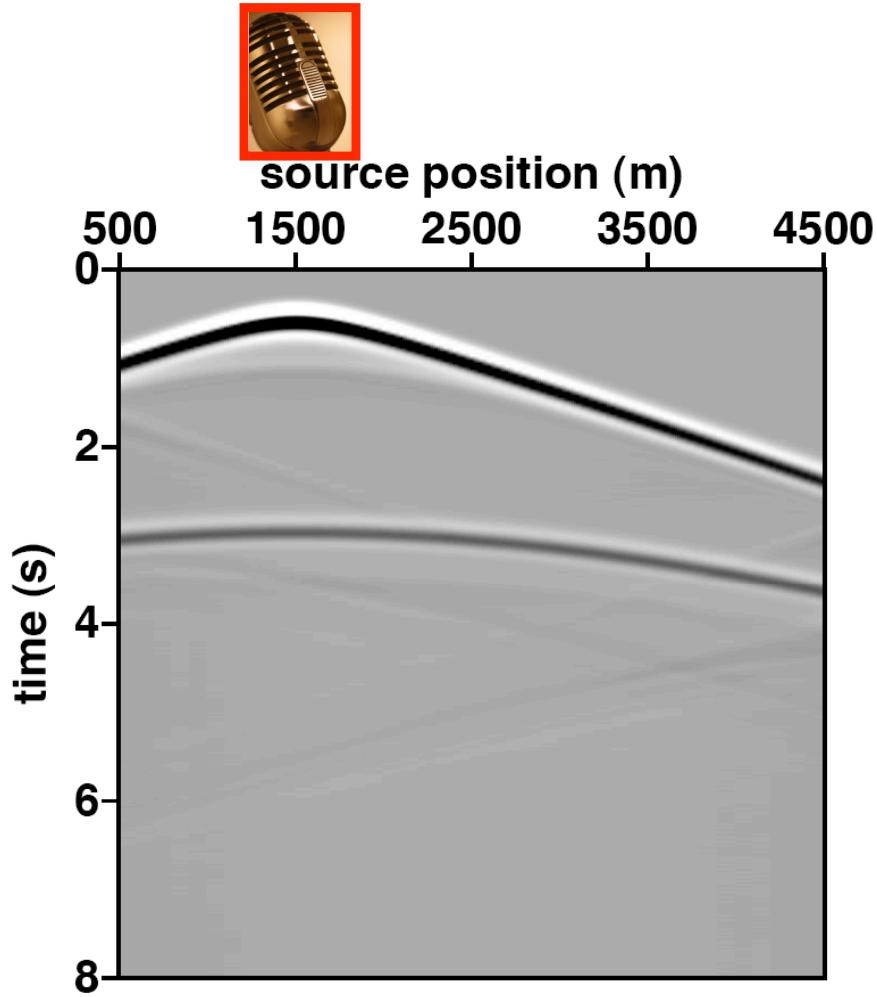
Boundary condition



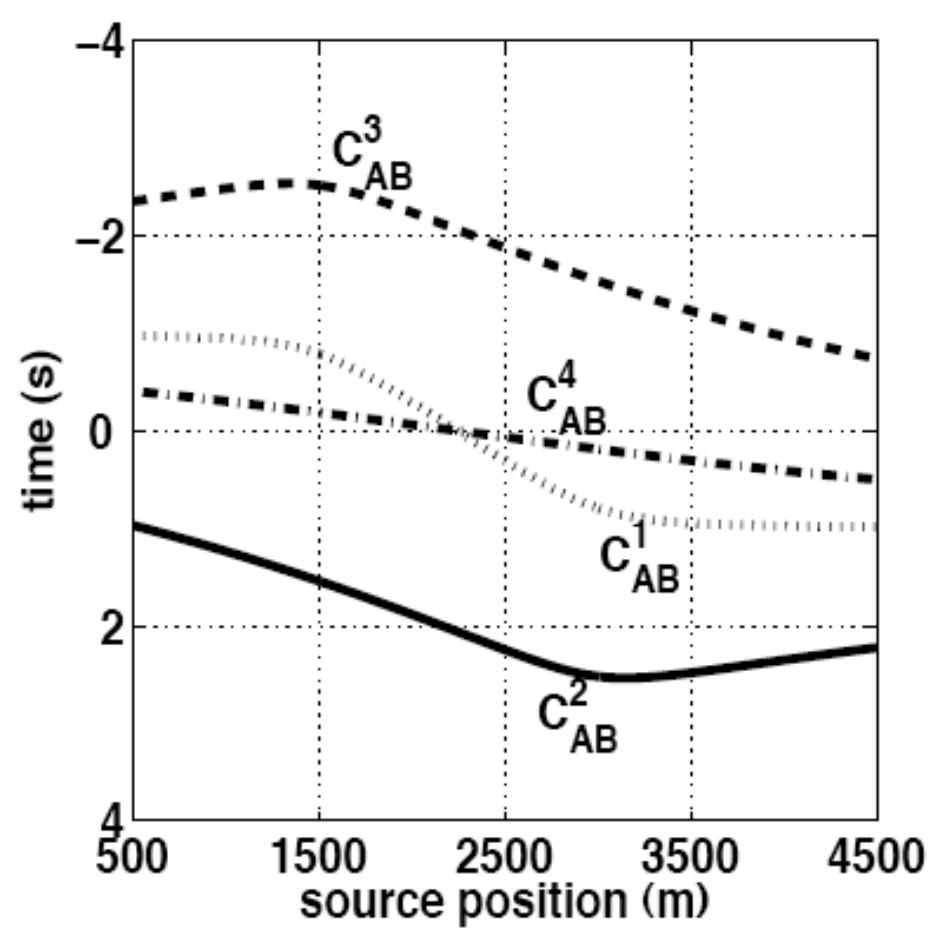
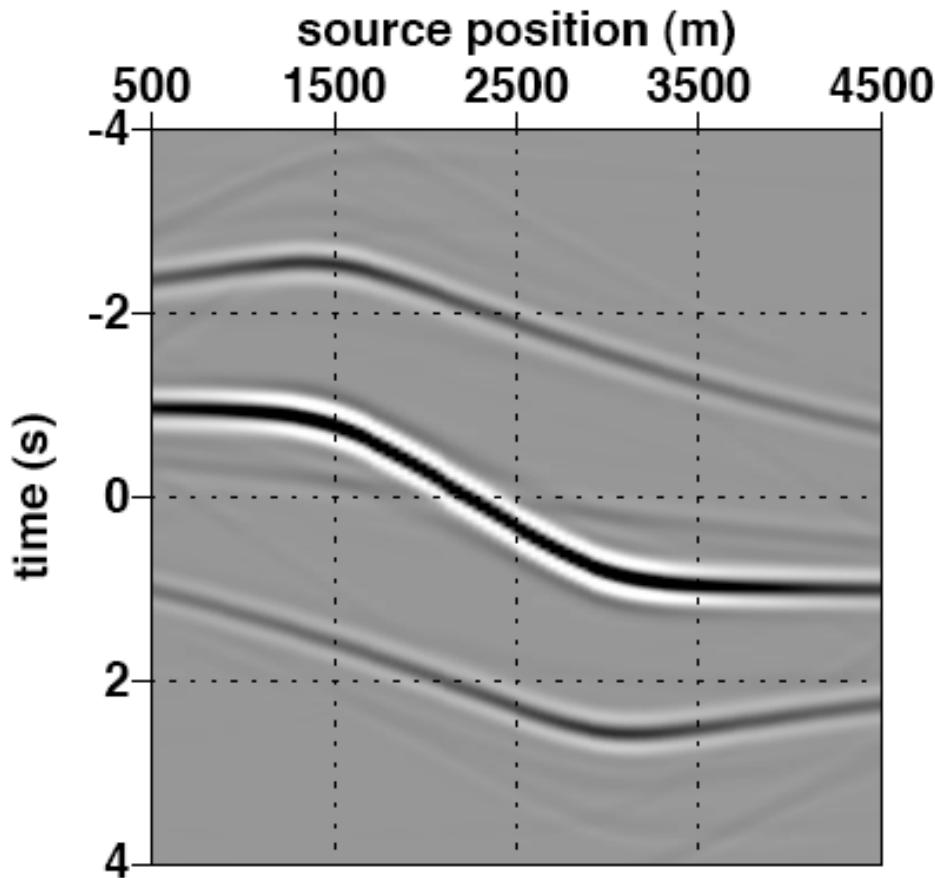
Model



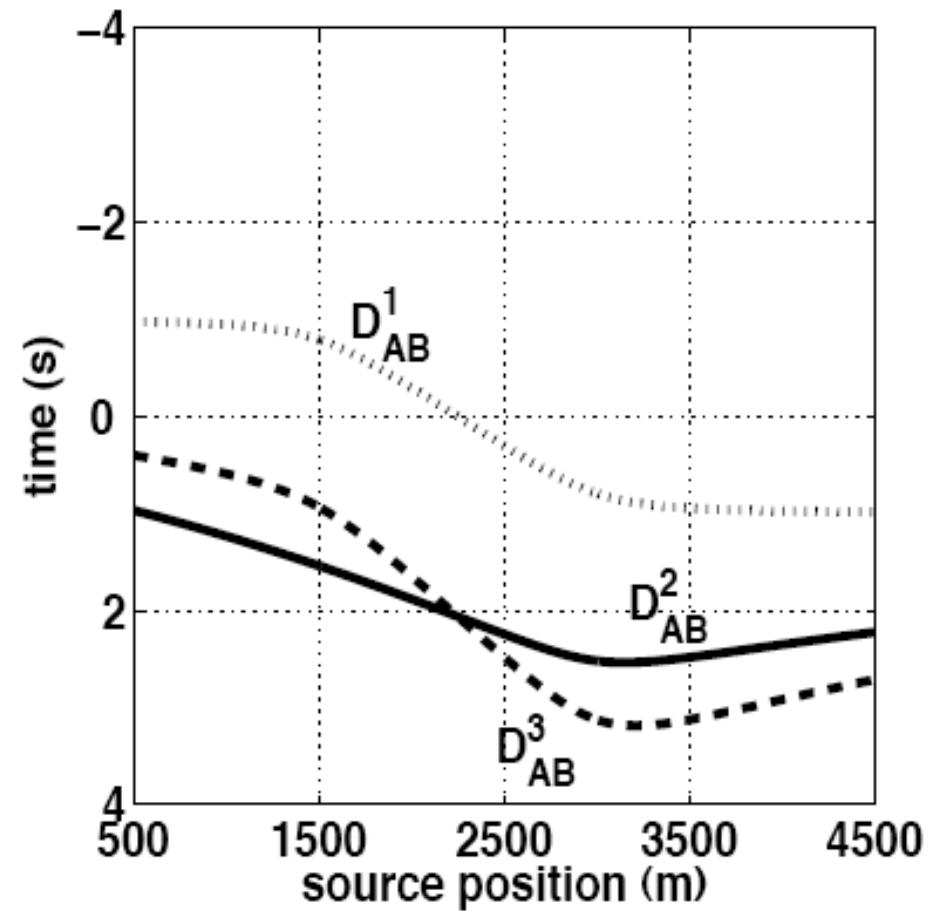
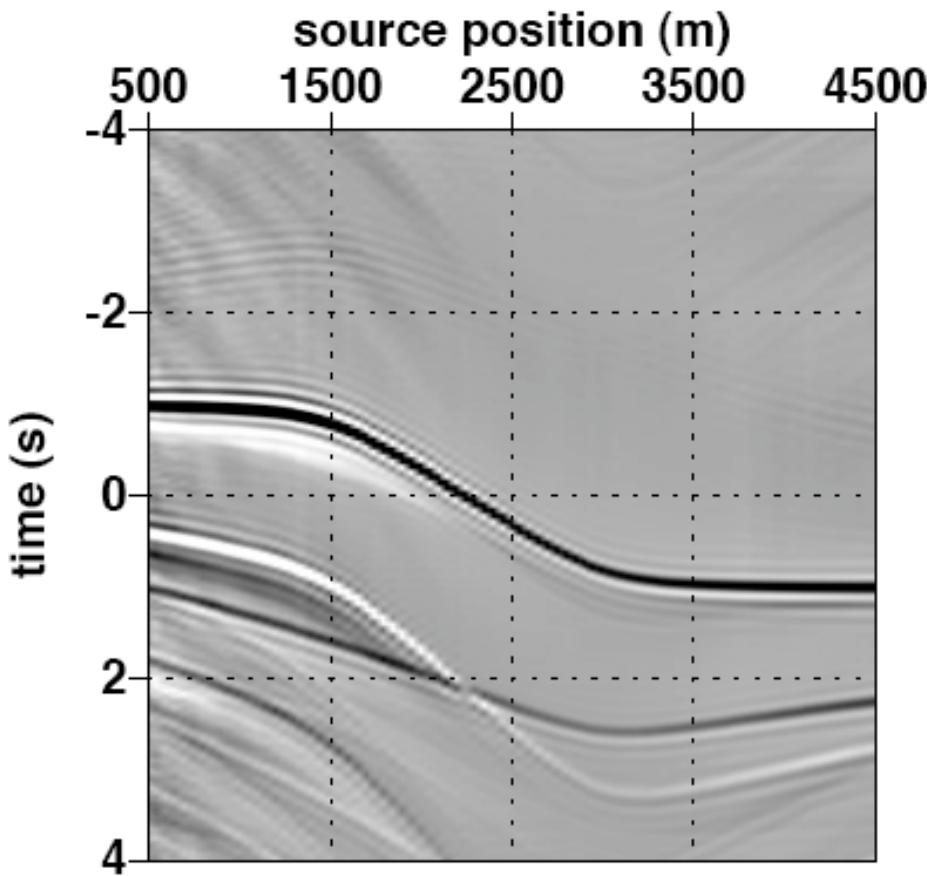
Data



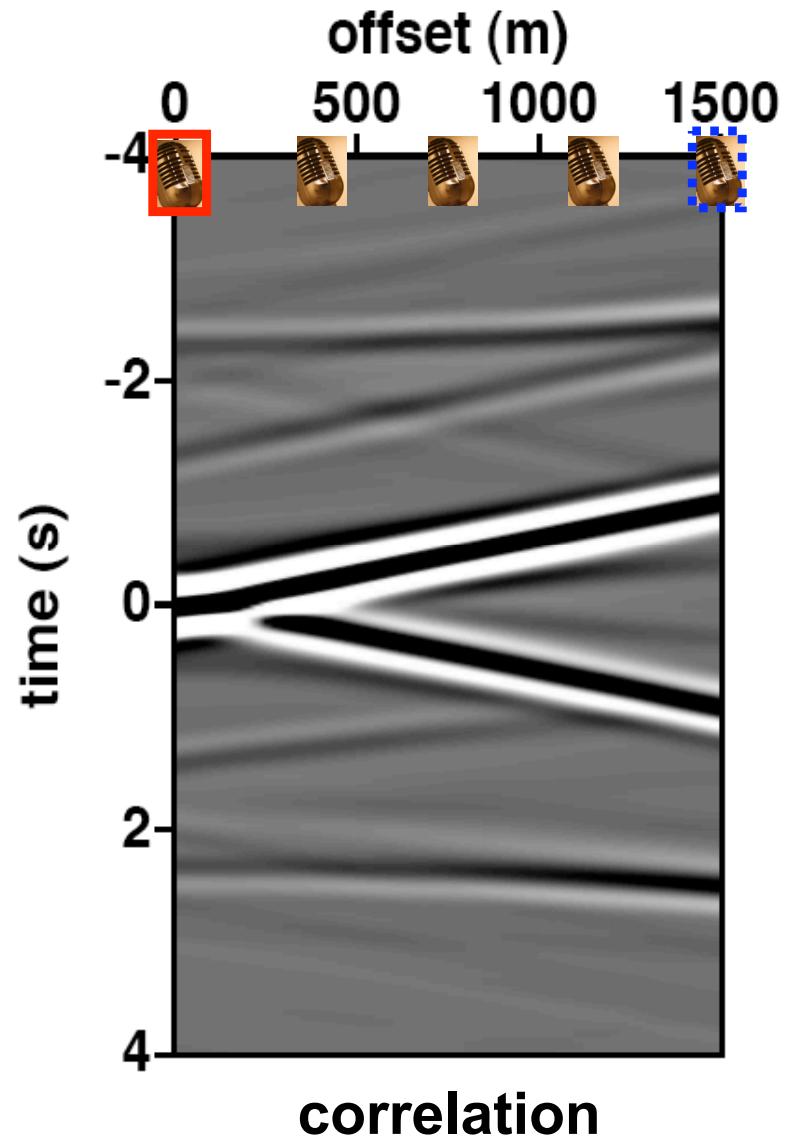
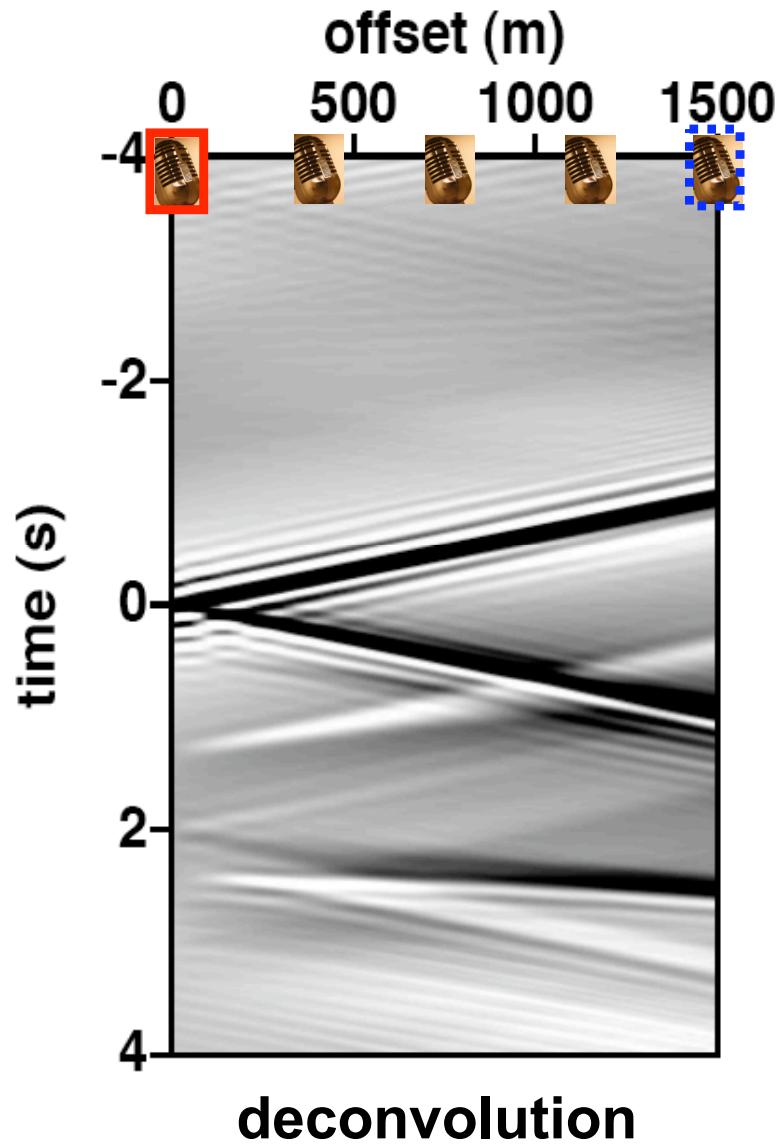
Correlation gather



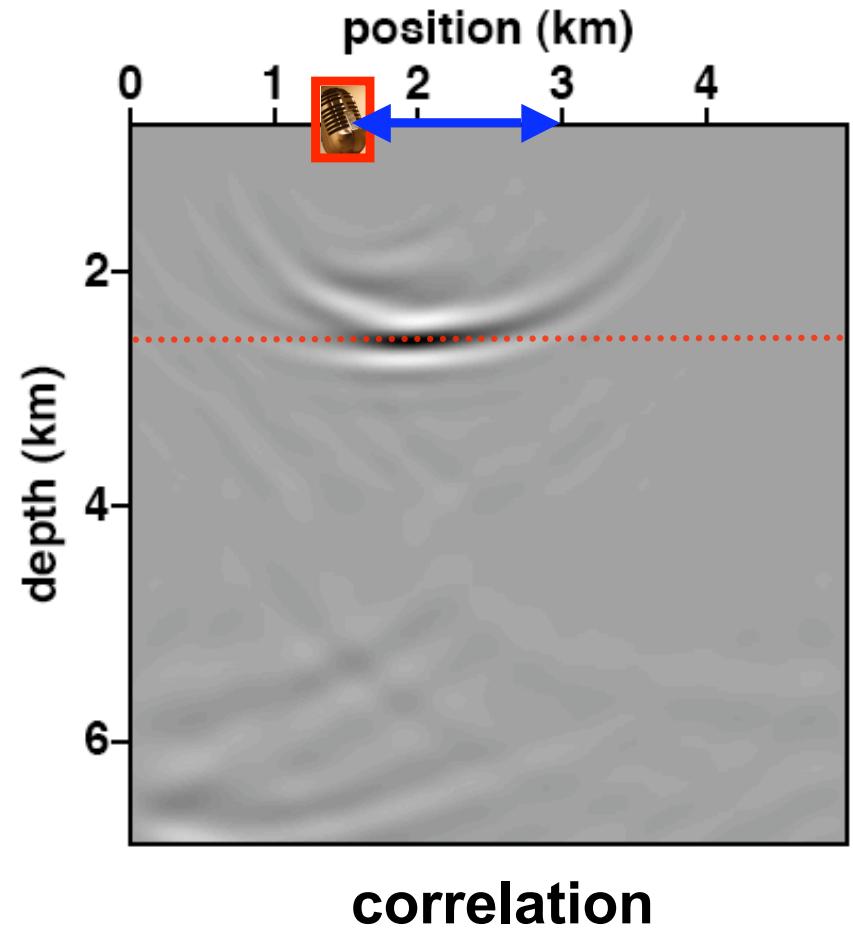
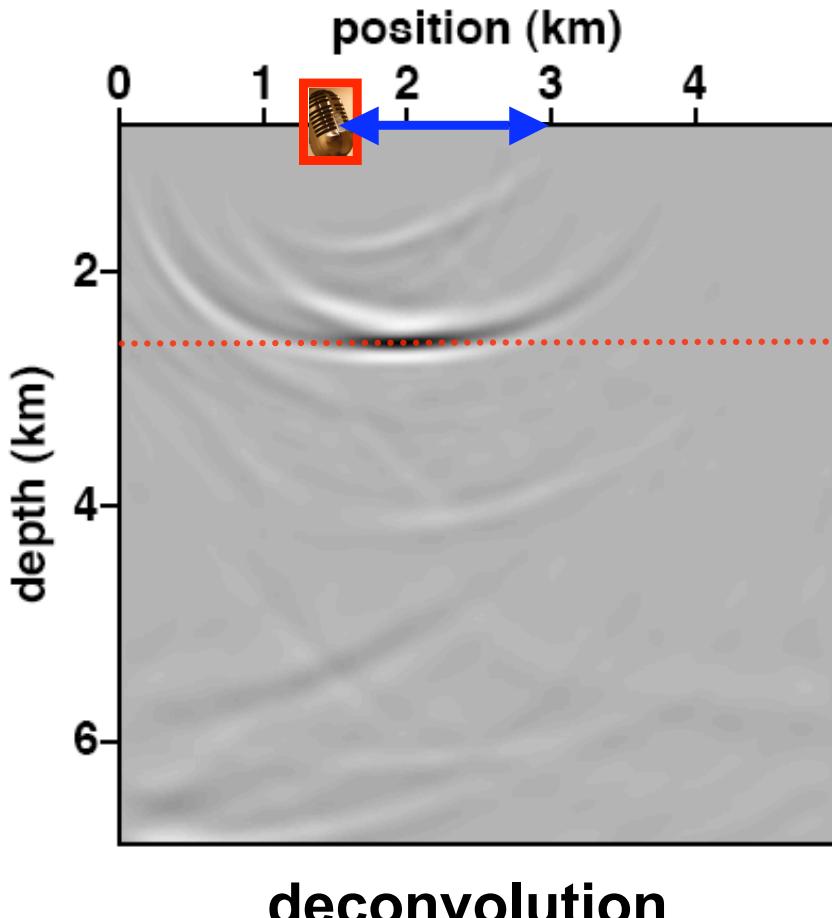
Deconvolution gather



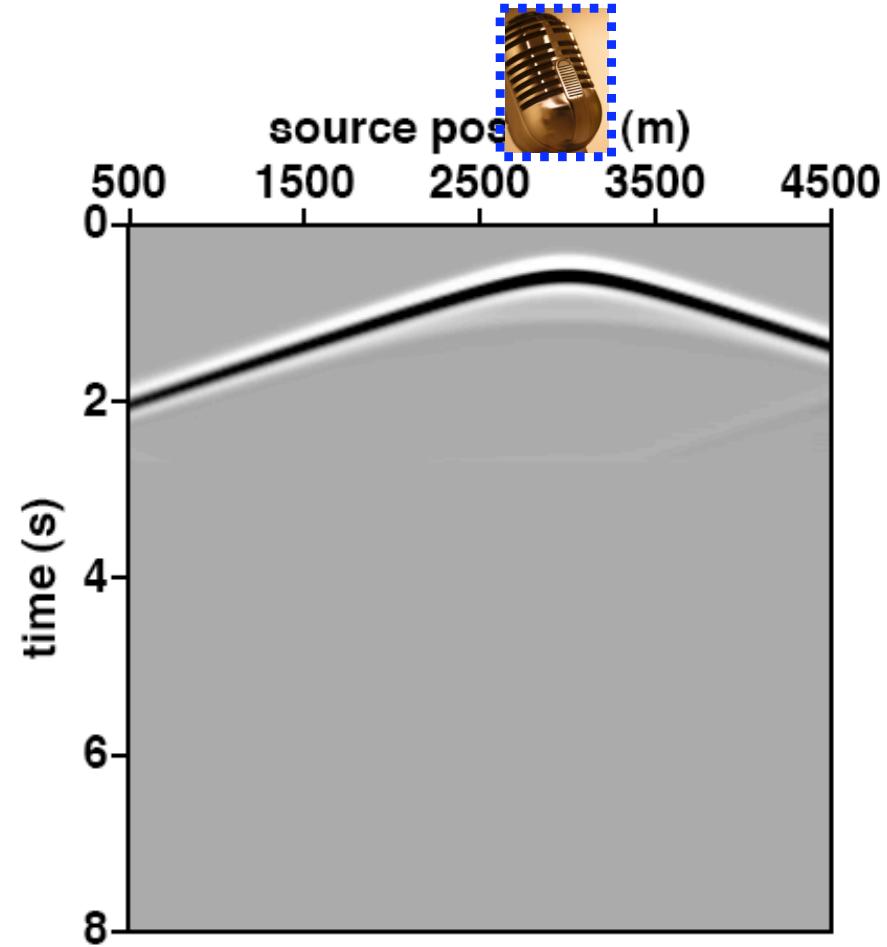
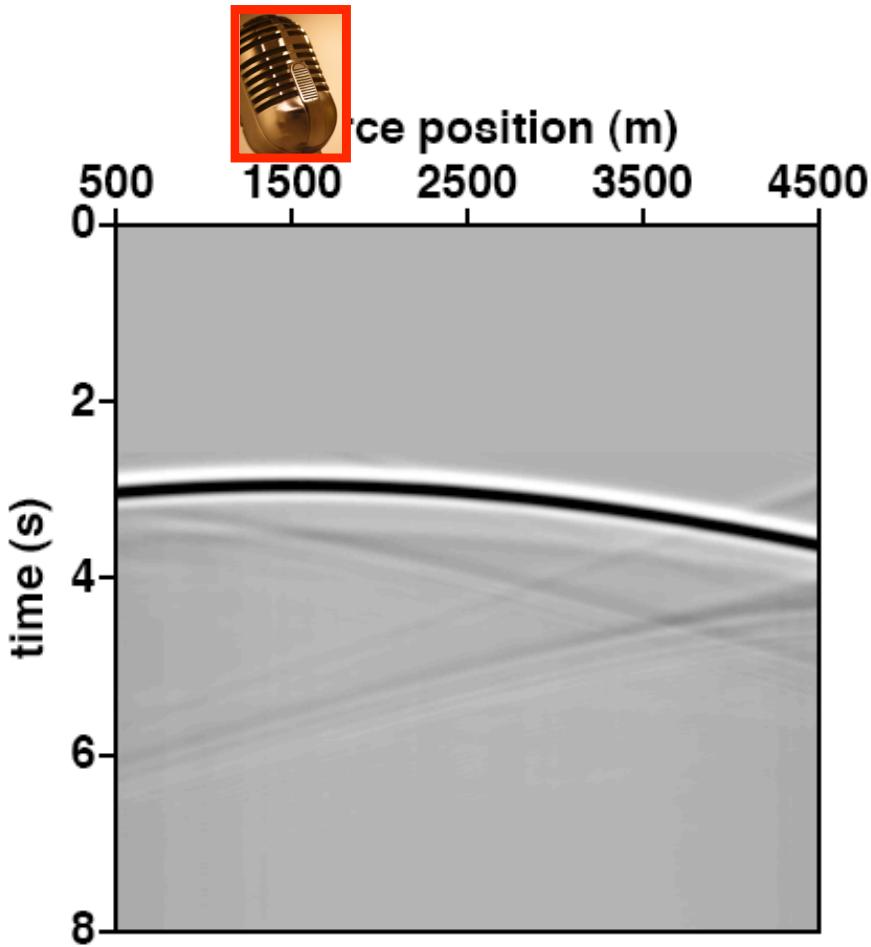
Pseudo-shot gathers



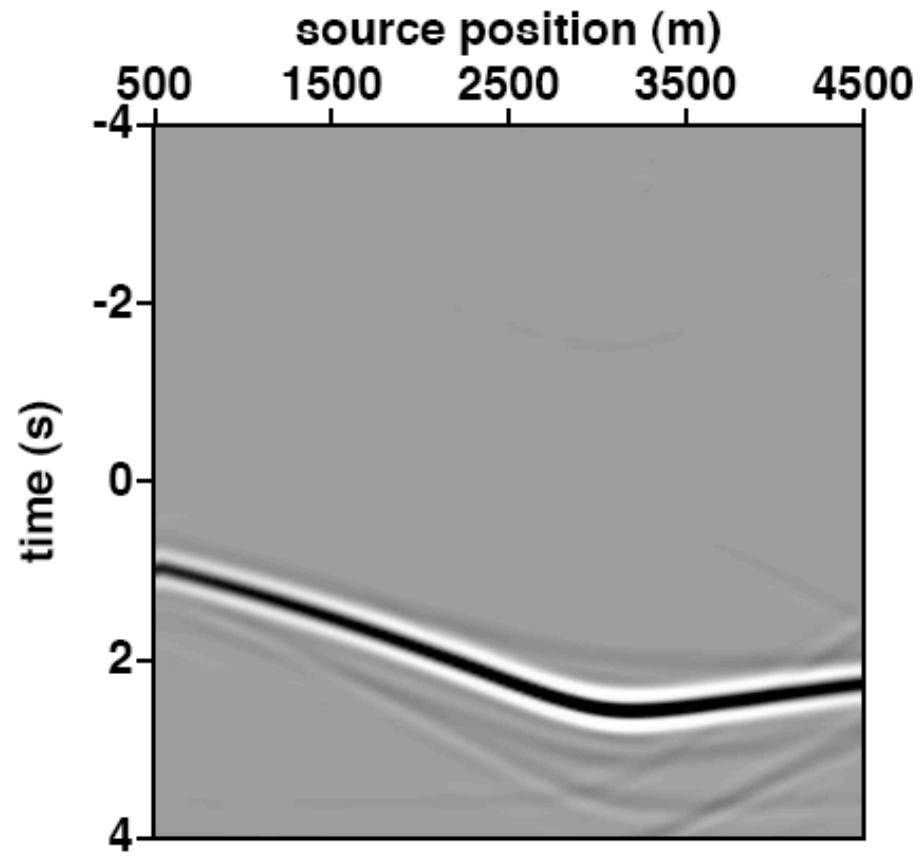
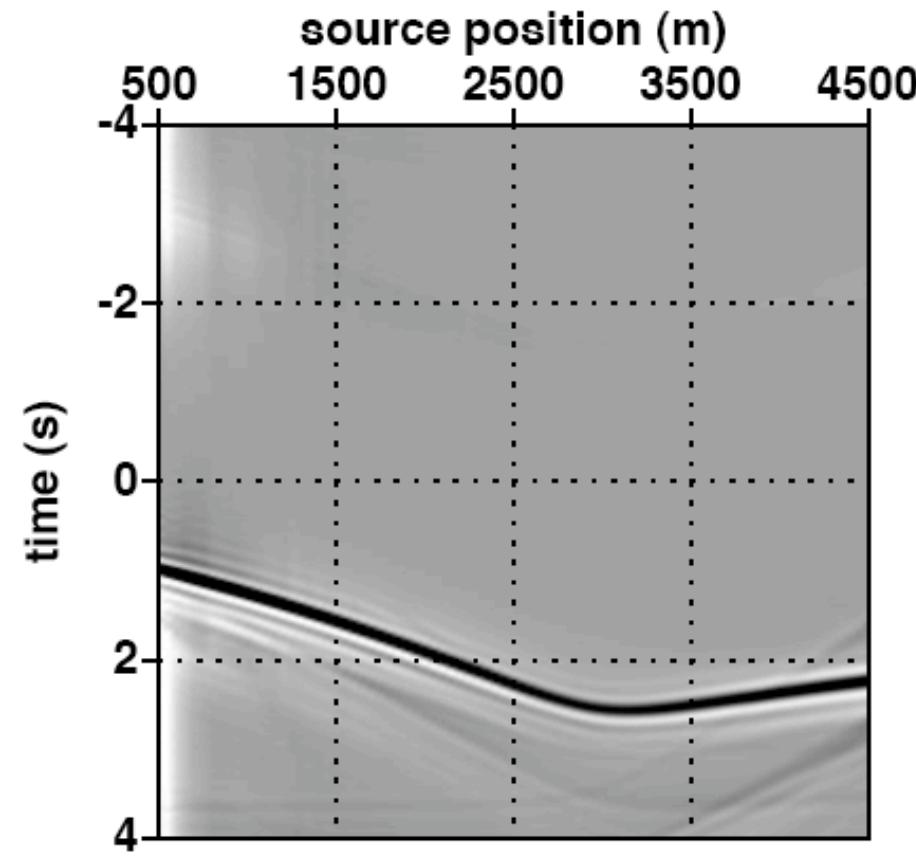
Shot-profile images



Wavefield separation



Only causal scattering





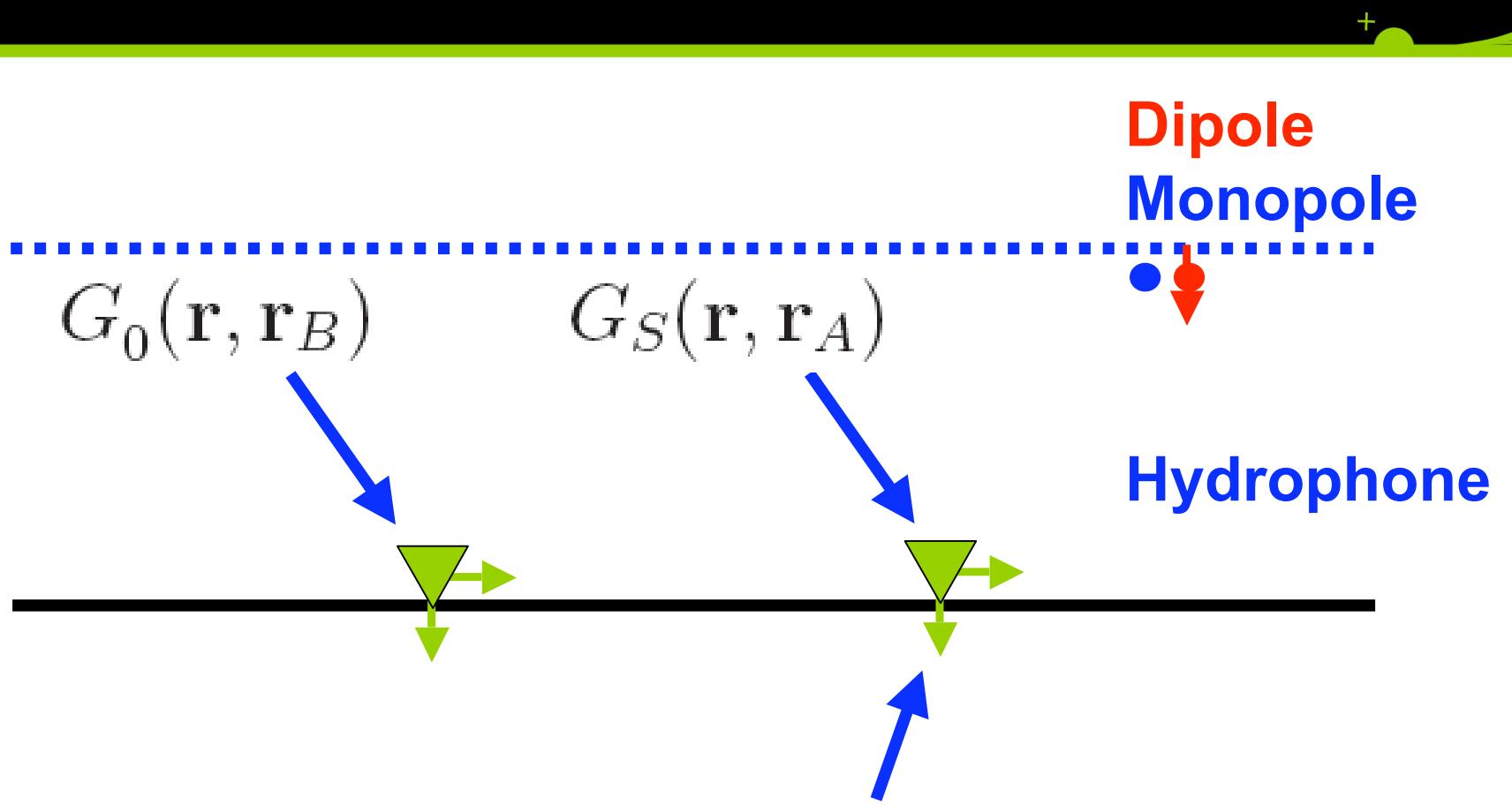
+

OBC (4-C)



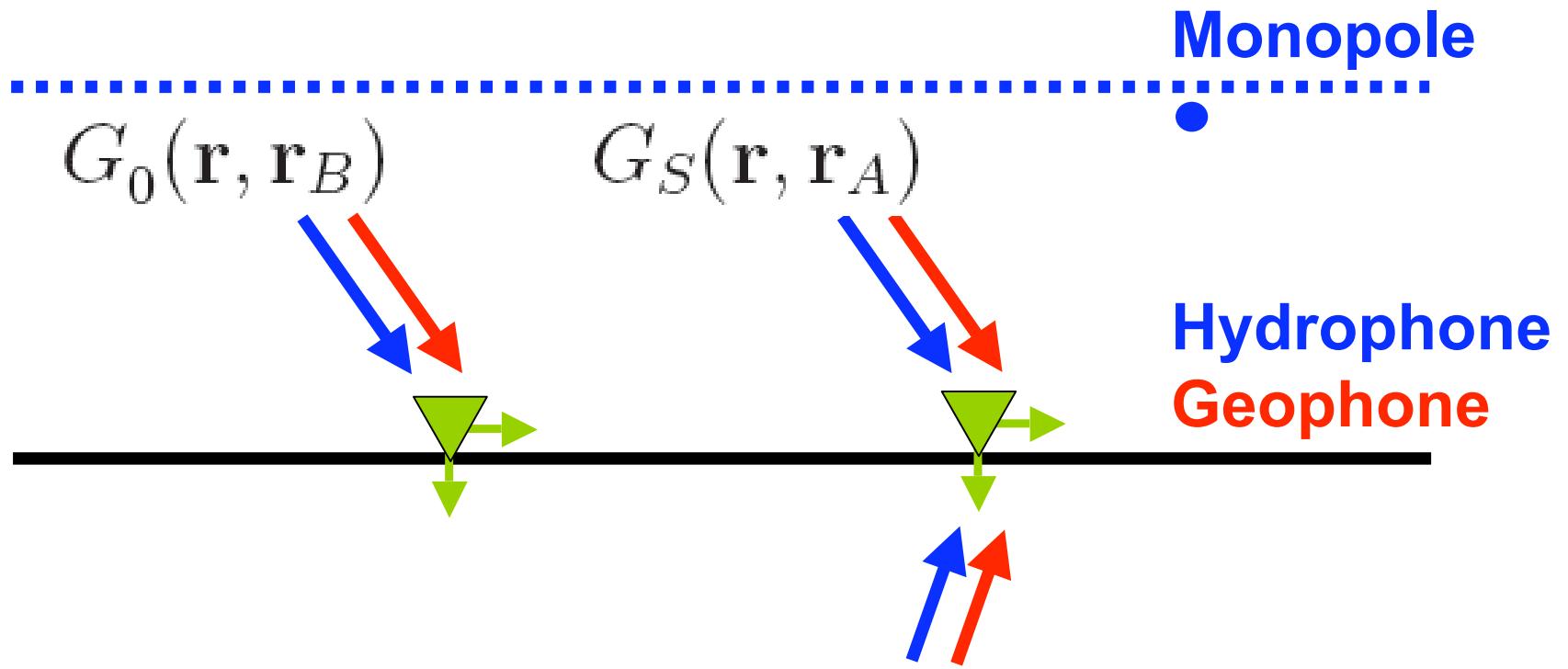
- imaging
- time-lapse (monitoring)
- CO₂ sequestration
- reservoir characterization

Dual-field Interferometry



$$G_S(\mathbf{r}_B, \mathbf{r}_A) = \oint_{\mathbf{r} \in \partial \mathbb{V}} \frac{1}{i\omega\rho} [G_S(\mathbf{r}, \mathbf{r}_A) \nabla G_0^*(\mathbf{r}, \mathbf{r}_B) + G_0^*(\mathbf{r}, \mathbf{r}_B) \nabla G_S(\mathbf{r}, \mathbf{r}_A)] \cdot d\mathbf{S}$$

Dual-field Interferometry

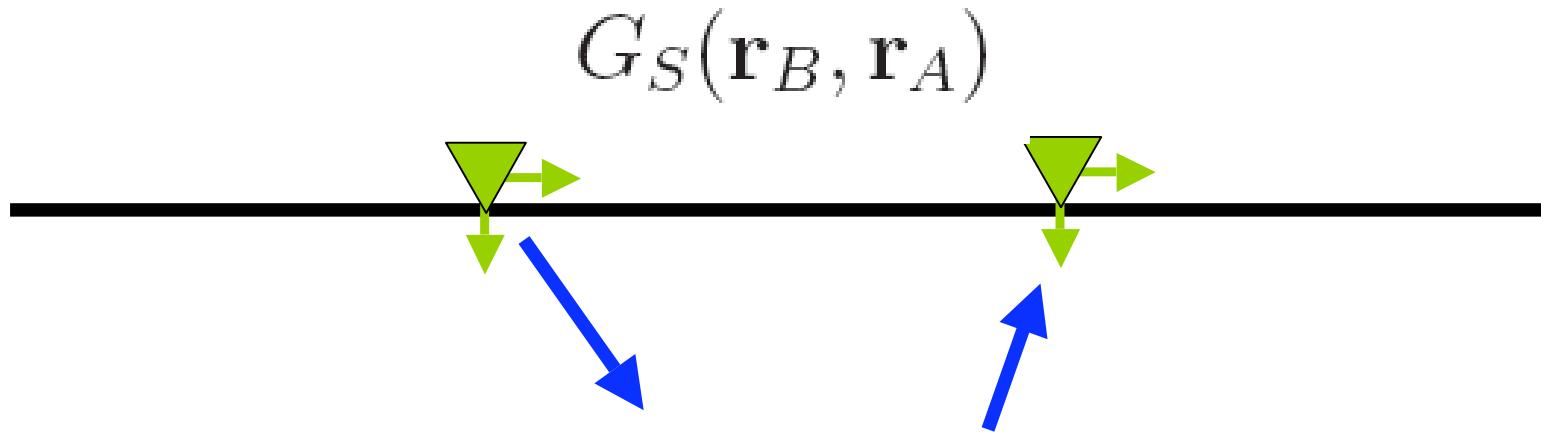


$$G_S(\mathbf{r}_B, \mathbf{r}_A) = \oint_{\mathbf{r} \in \partial \mathbb{V}} \frac{1}{i\omega\rho} [G_S(\mathbf{r}, \mathbf{r}_A) \nabla G_0^*(\mathbf{r}, \mathbf{r}_B) + G_0^*(\mathbf{r}, \mathbf{r}_B) \nabla G_S(\mathbf{r}, \mathbf{r}_A)] \cdot d\mathbf{S}$$

Dual-field Interferometry



No free surface!!!

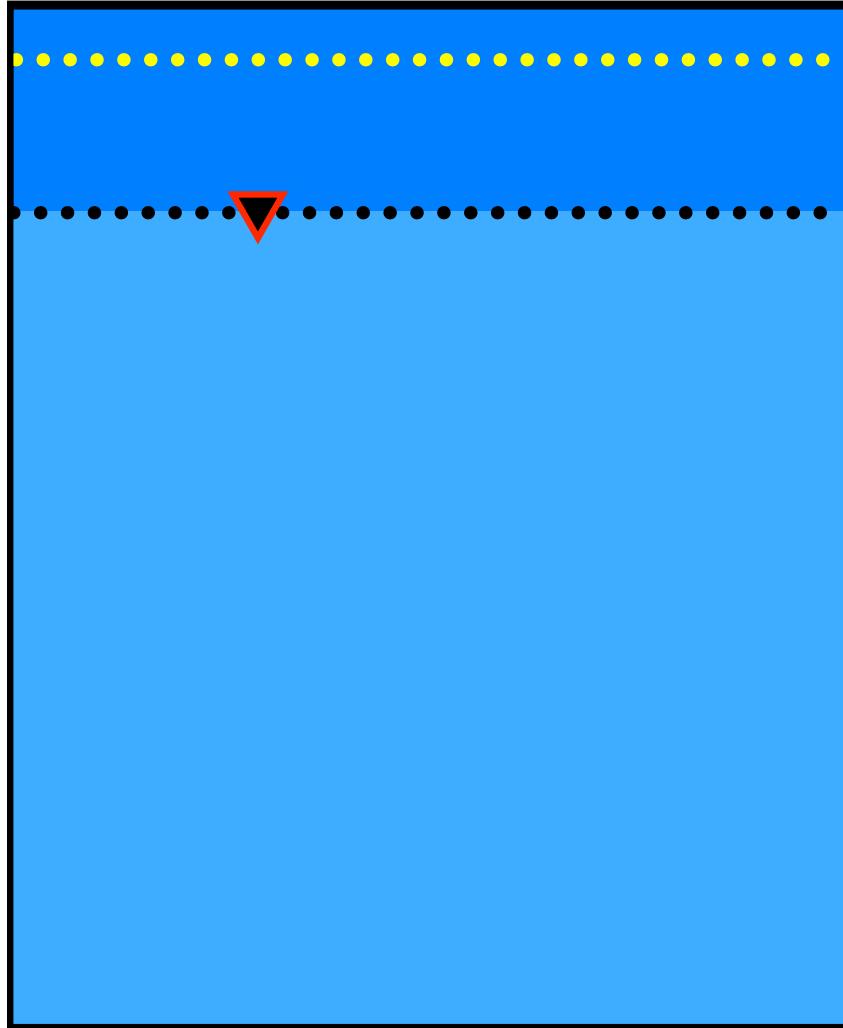


See also Mehta et al., 2007, Geophysics;
Wapenaar et al., 2008, Geophys. Prosp.

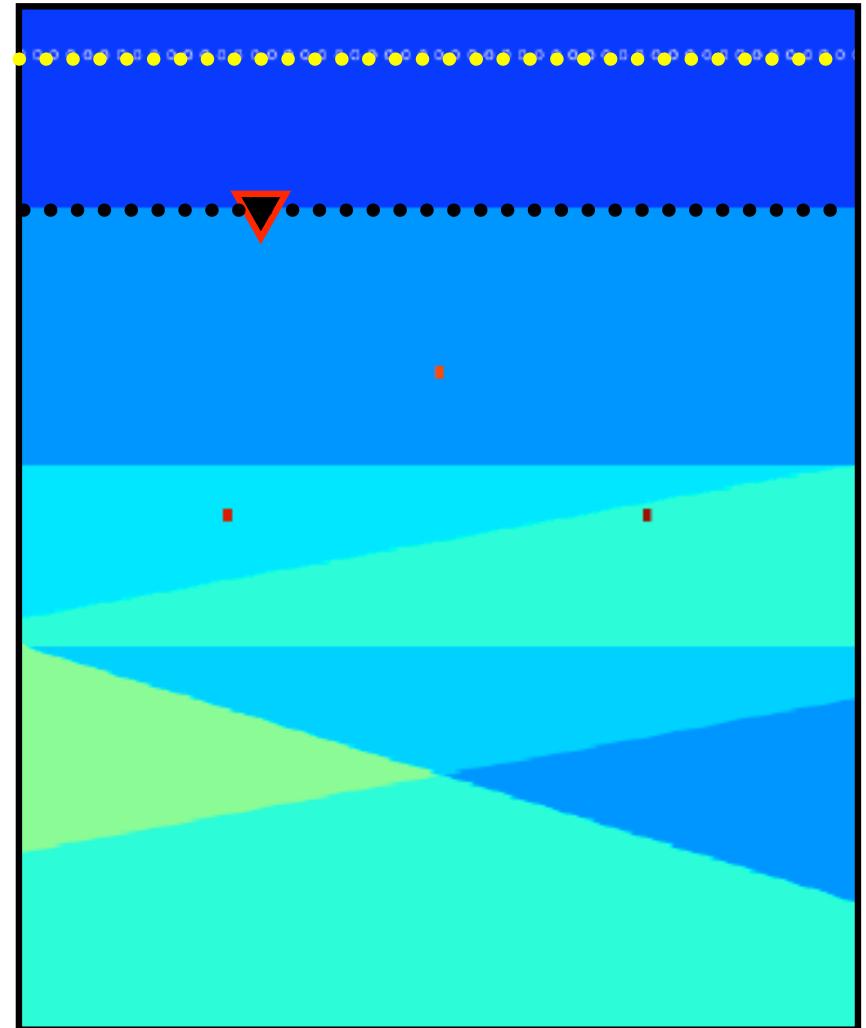
Acoustic OBC model



Reference



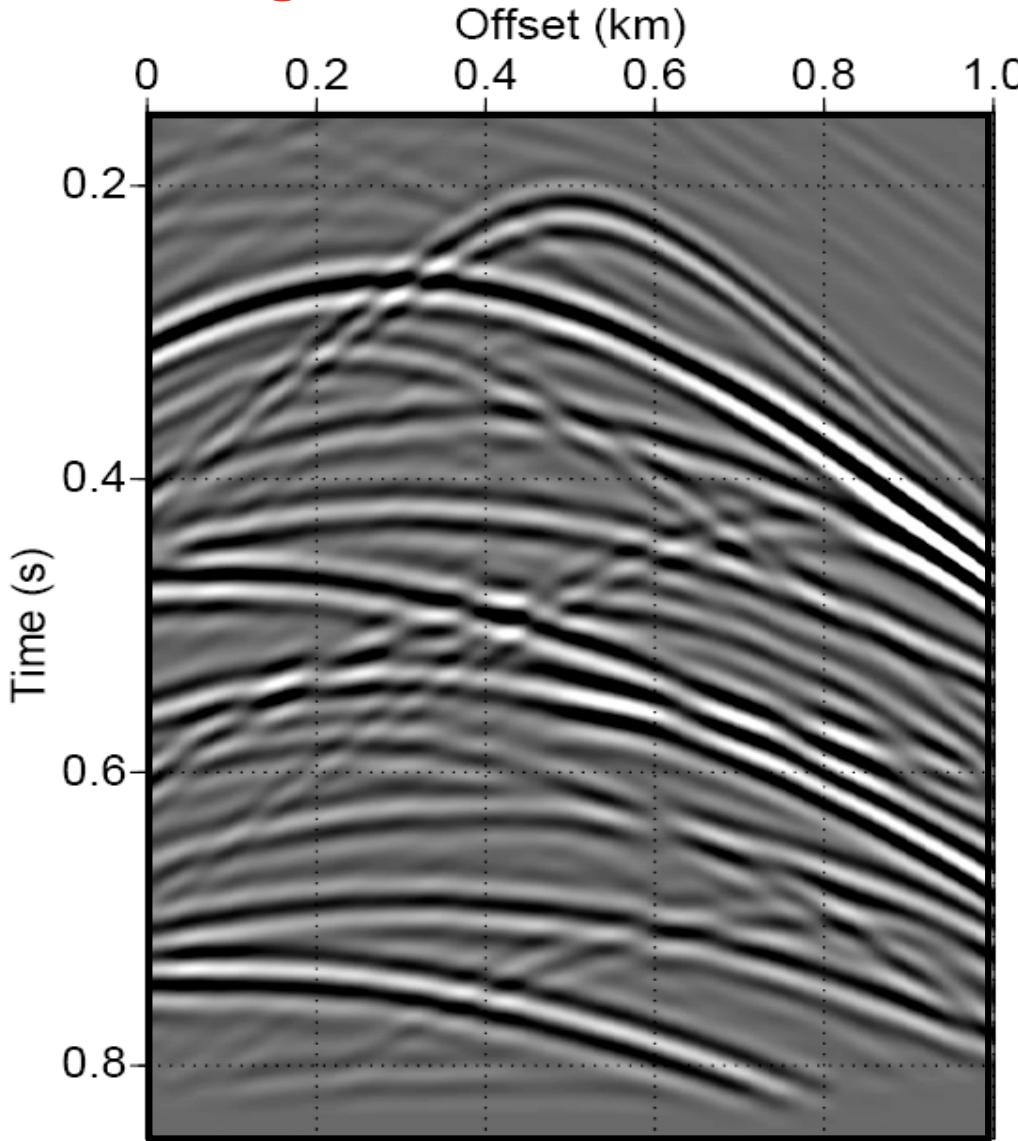
Perturbed



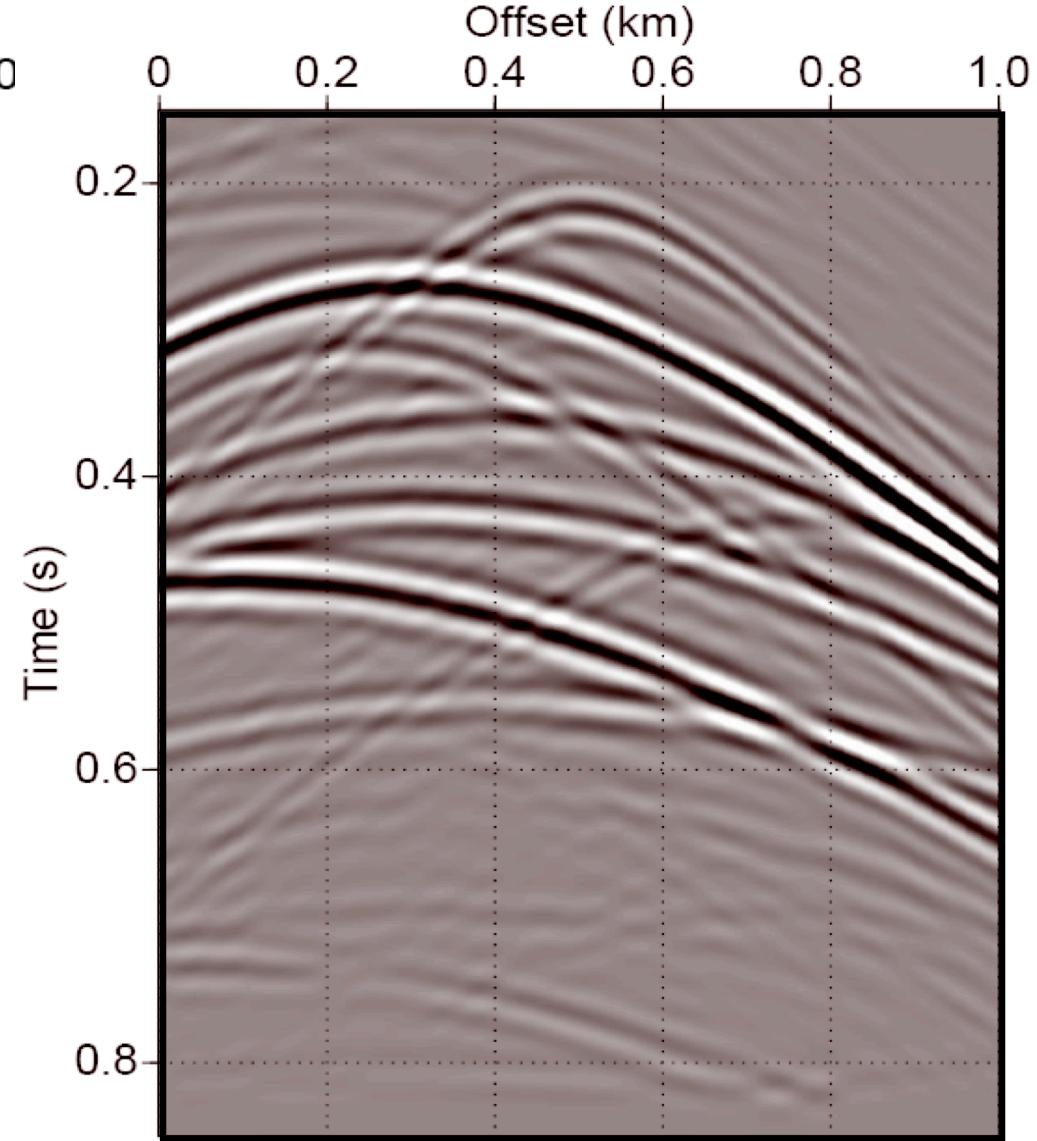
Pseudo-shot gathers



Single-field; free surface



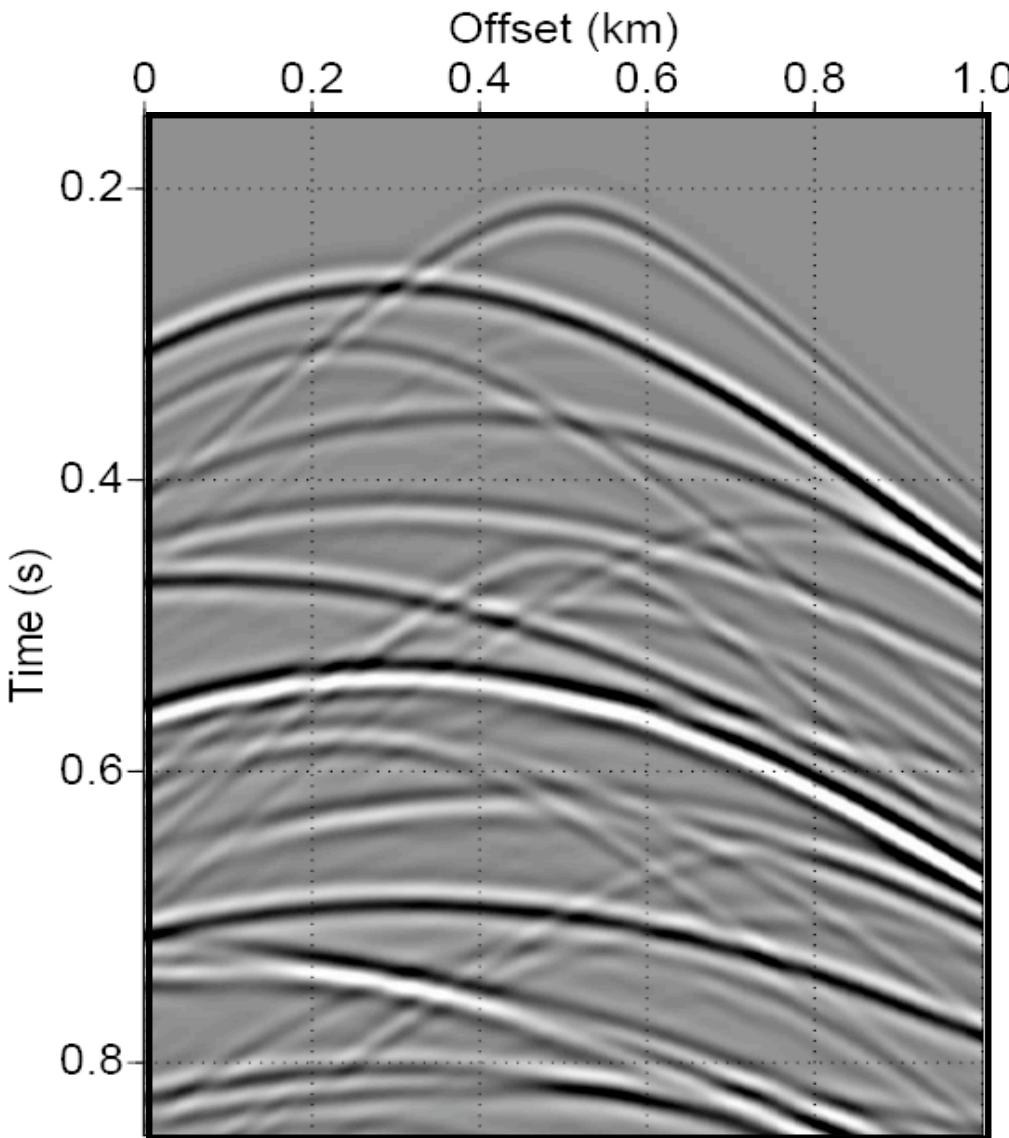
Dual-field; free surface



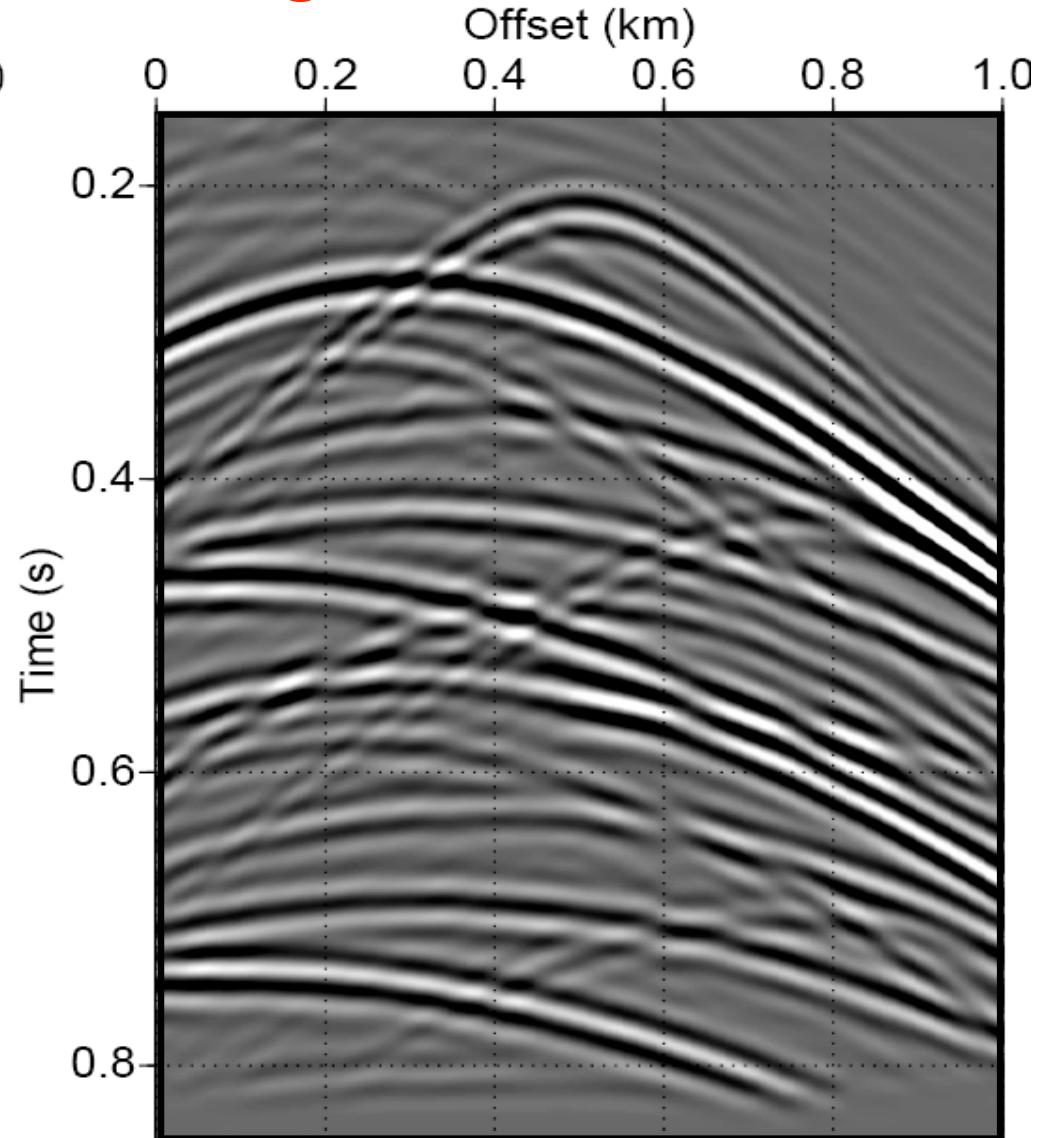
True response vs. single-field



True; WITH free surface



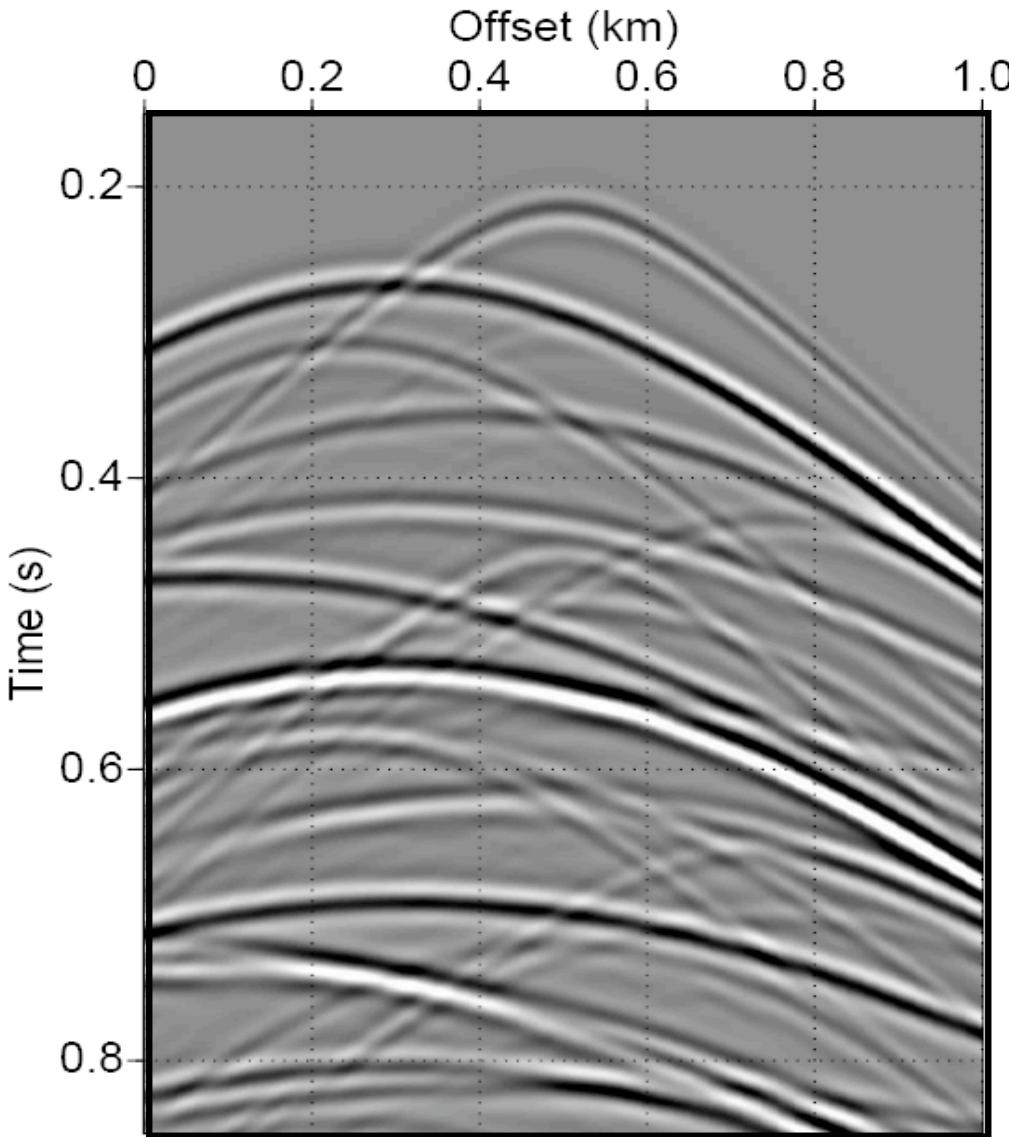
Single-field; free surface



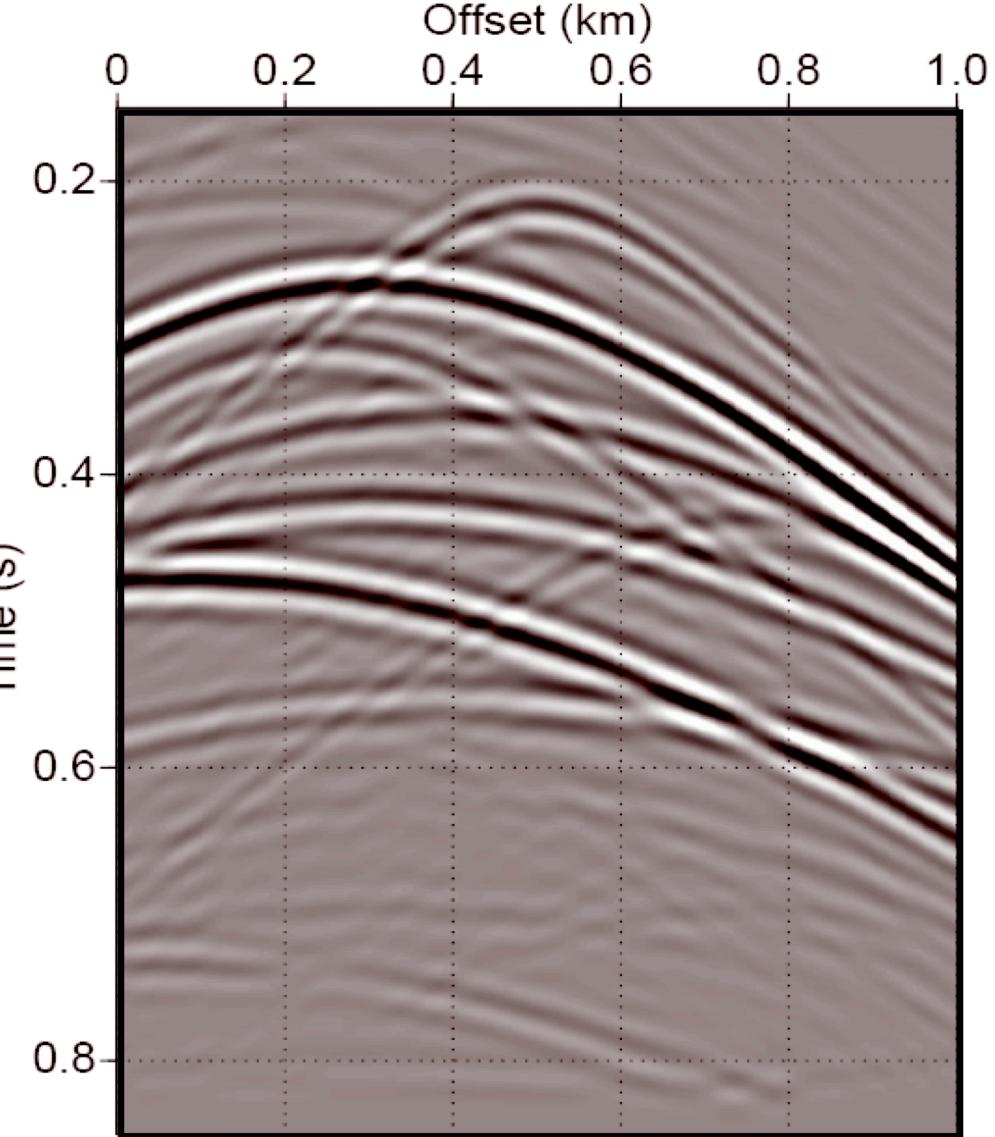
True response vs. dual-field



True; WITH free surface



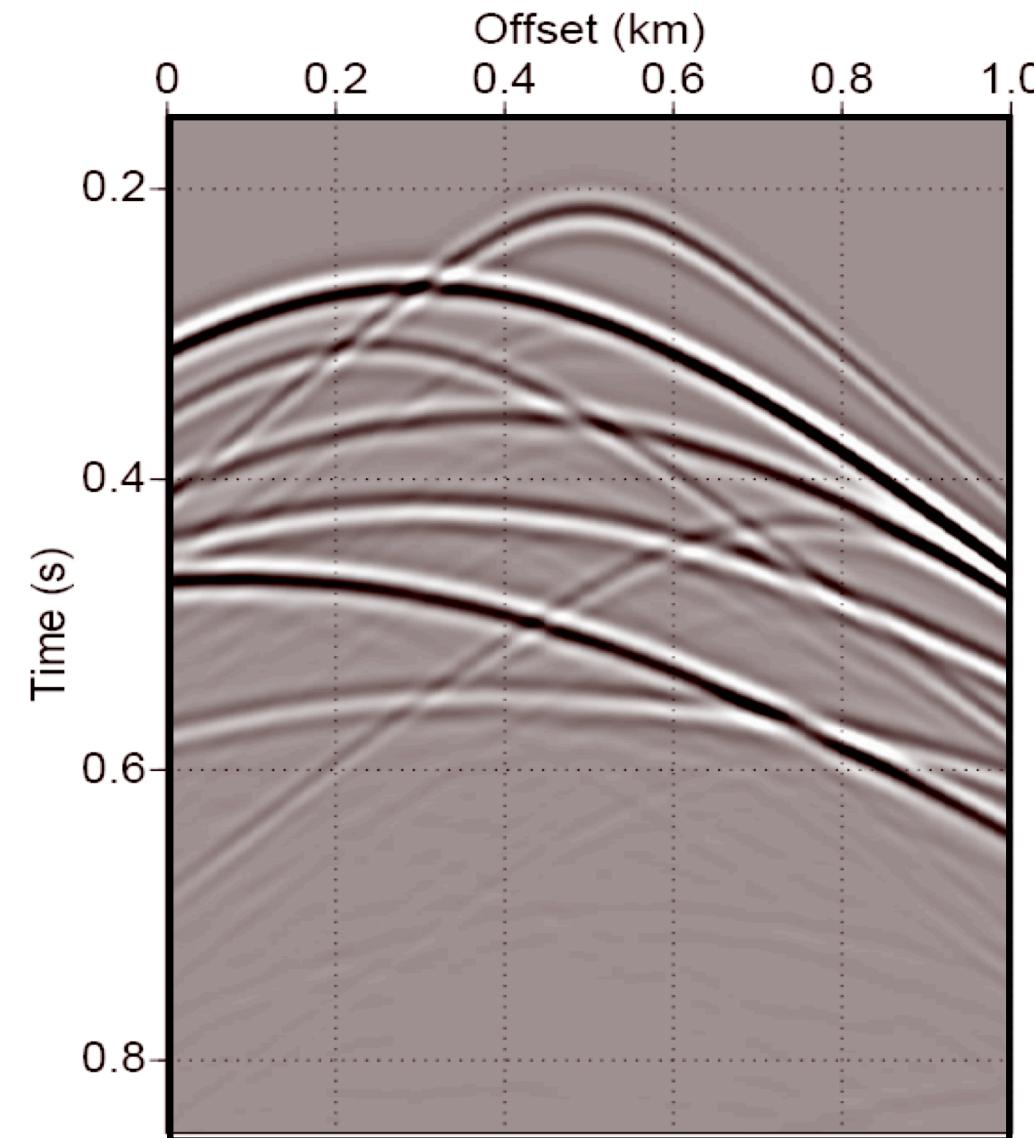
Dual-field; free surface



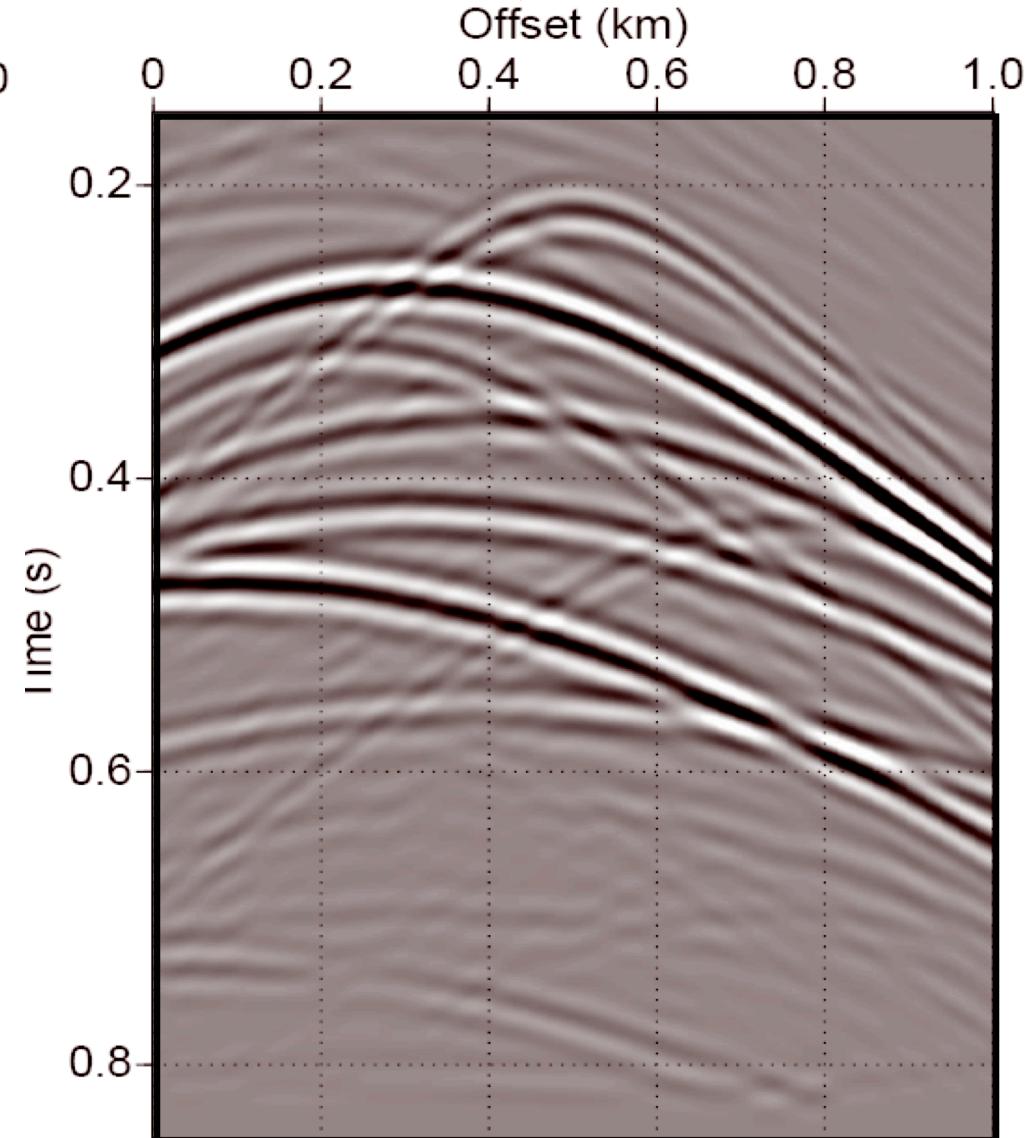
True response vs. dual-field



True; WITHOUT free surface

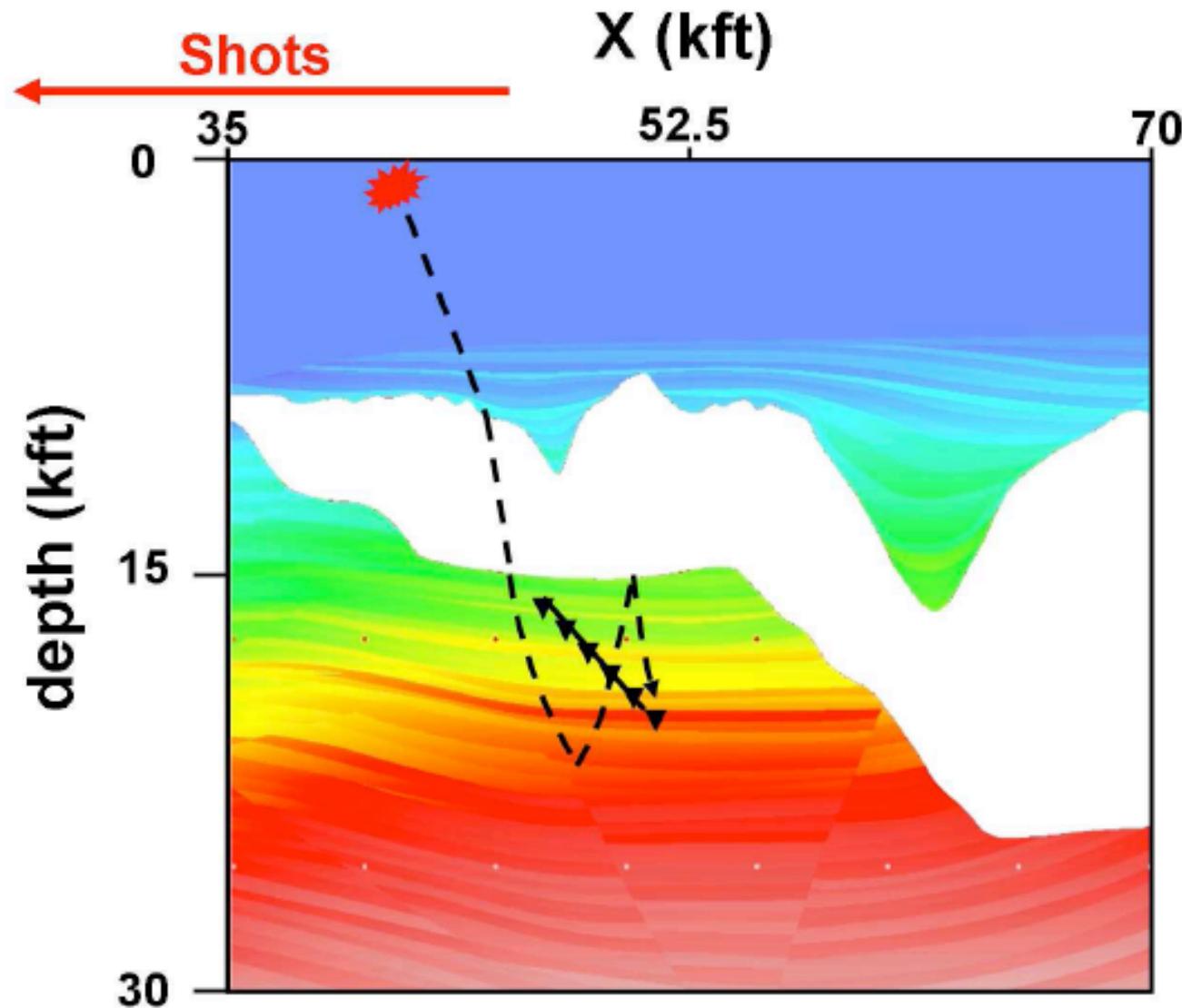


Dual-field; free surface

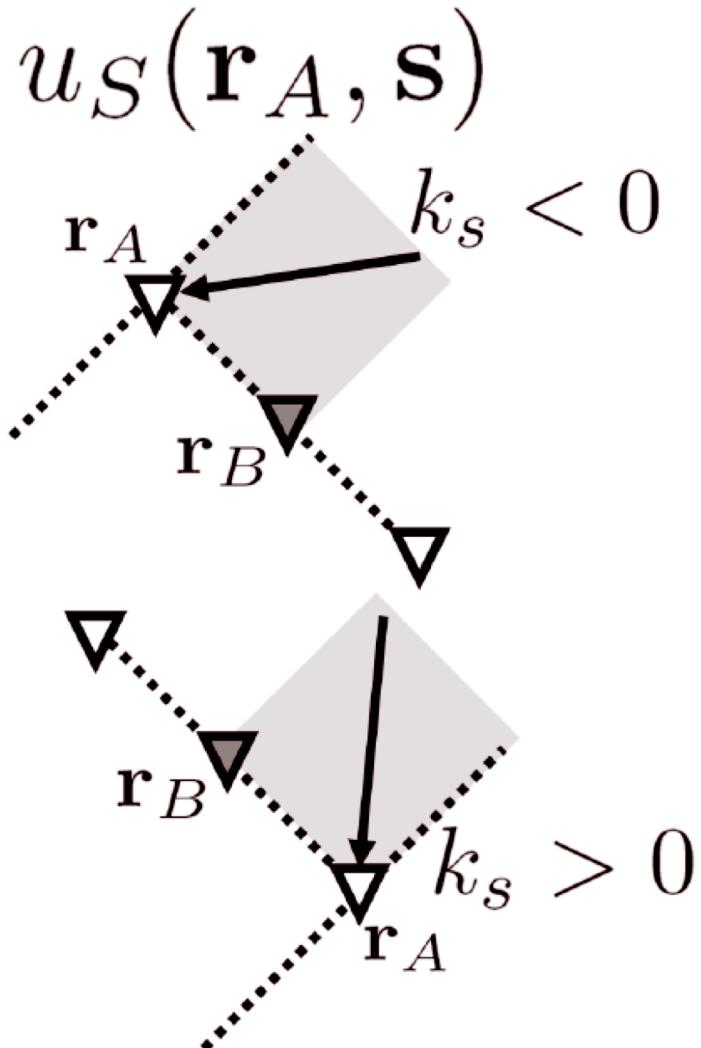
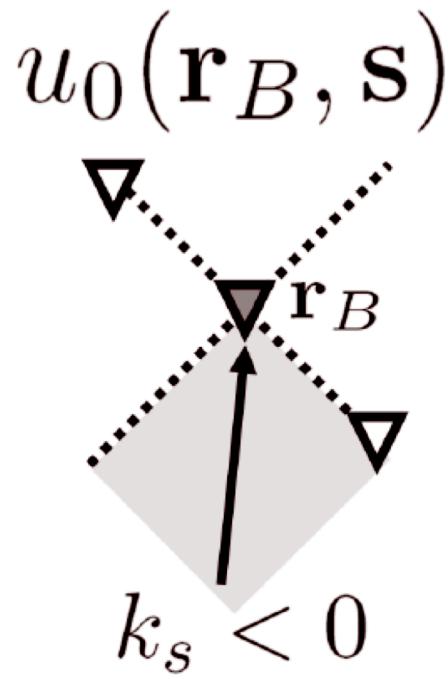


+

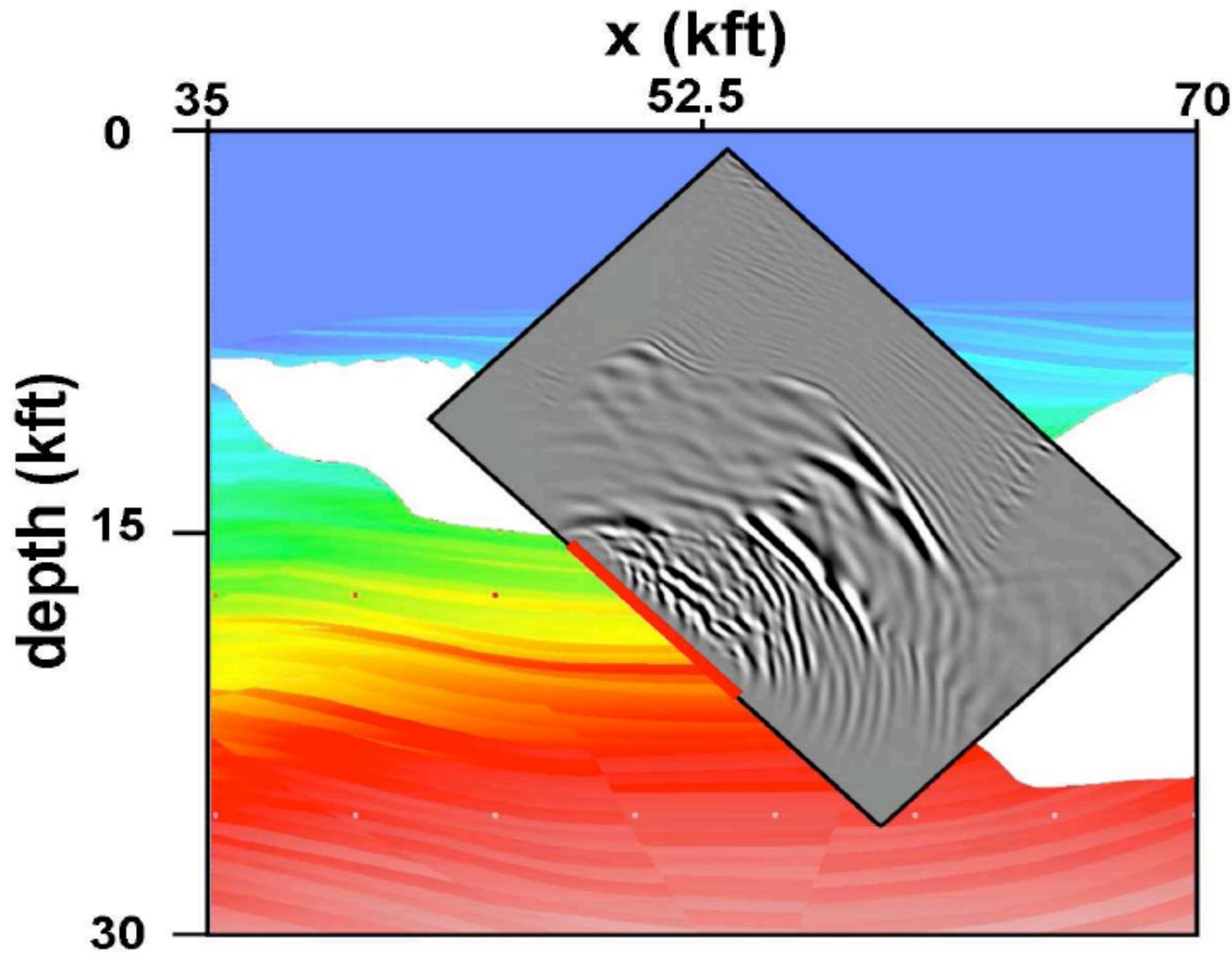
Imaging multiples



Wavefield separation



No wavefield separation



Imaging multiples: with separation

